

# A LESLIE-TYPE URBAN-RURAL MIGRATION MODEL, AND THE CASE OF TURKEY\*

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## Abstract

Economic growth and urbanization are often closely linked. From a formal point of view, urbanization can be seen as a result of migration probabilities, together with fertility rates and survival probabilities prevalent in urban and rural areas, together with migration probabilities. We introduce a new Leslie-type population model which describes the age structure of urban and rural populations and allows for migration in both directions. This model can project the current population structure into the future and also permits an analysis of the long-run properties of the population such as future urbanization under the assumption that the current conditions will persist. As an example, we study the population structure of Turkey with respect to fertility, survival probability, and migration rates. We find that the current Turkish population, when compared to its stable counterpart, is younger and its urbanization is higher. Our model permits an assessment of the impact of migration policies on the future and long-run urbanization of Turkey.

**Key words:** Leslie-type model; migration; stability; ageing; Turkish population; urbanization; population growth

## 1 Introduction

“The world is undergoing the largest wave of urban growth in history”, the United Nations Population Fund warns in an 2007 online release.<sup>1</sup> Meanwhile, world urbanization has arrived at the 50% level, with a projected annual growth rate of 1.9%, which is an aggregate of values spanning from 0.7 for more developed regions to 4.0 for least-developed countries (UNFPA State of World Population report 2010 [14]).

Urbanization and economic growth are often closely linked, but urbanization as well concentrates poverty. On the other hand, the increasing urbanization is insofar problematic as it

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<sup>1</sup><http://www.unfpa.org/pds/urbanization.htm>, accessed: 2011-05-14

contributes to the aggravation of the structural weakness of rural regions which may be a major push factor of rural-urban migration. Urbanization may also be linked to decreasing fertility and shrinking, hence ageing populations, as there is a gap in fertility levels between urban and rural regions throughout the world.<sup>2</sup>

With a 70% proportion of population living in urban areas today, Turkey is on a level with more developed regions (75%), while its projected urban growth rate of 1.9% is characteristic for less-developed countries as described by the 2010 UNFPA report [14]. Urbanization in Turkey started on a large scale in the early 1950s from a level of 20% only. For the Turkish population between 1955 and 2000, Gedik [5] investigated the Alonso theory of differential urbanization which postulates cycles of three evolutionary phases, urbanization, polarization reversal, and counter urbanization, where growth rates are highest for large, medium, and small settlement sizes, respectively. Gedik found evidence for a phase of pre-concentration in small cities in the 1950s, large-city urbanization that followed, and polarization “dispersal” starting in 1980 with highest growth rates in medium-sized cities dispersed throughout the country.

Simultaneously, Turkey experienced a pronounced change in fertility. Within three decades, the total fertility rate halved down to barely 2 today (2.09 in 2010, c.f. [14]). The Turkey Demographic and Health Survey 2008 [7] investigates fertility preferences and behaviour of Turkish women by residence, and provides information on maternal and child health. Among the findings is that, though total urban fertility is below replacement and the urban-rural differential appears to be contracting over time, clear above-replacement levels in South and East Anatolia persist. At each age class, rural women tend to bear more children than women in urban centers where a trend that fertility decreases with a higher educational level can be observed. As a result, the urban-rural gap with respect to the median age at first birth is broadening. A significantly higher proportion in urban centers of women in the working ages certainly adds to this gap, and thus the effects of rural-to-urban migration of economically active women. Furthermore, the findings of the survey indicate a significant urban-rural differential in child mortality which appears to be correlated with the mother’s young age and educational level.

From a formal point of view, the phenomenon of urbanization can be analyzed as a result of migration probabilities and their interaction with urban- and rural-specific fertility rates and survival probabilities. The approach may vary in several basic respects: The focus may be either on forecasting the future population using forecasts of mortality, fertility and migration, or on population projection in order to find answers to what the population would be like in the long run if mortality, fertility and migration would evolve (or persist) in a certain way. The effects of demographic and environmental conditions on the dynamics of populations may be studied in discrete or continuous time, in a deterministic framework, or using a probabilistic model with random variation in births, deaths, and migration.

In her overview of probabilistic approaches to demographic and population forecasting, Booth [2] identifies three widely-used frameworks: methods on the basis of sample data on individual expectations about future developments or expert opinions, structural modeling methods based on theories on relations between demographic variables and processes, and extrapolative methods using time series models to detect patterns and trends in the past and extrapolate them into the future. A time series approach recently adopted by Hyndman, e.g. [8], involves functional data models to forecast mortality, fertility, and migration.

The Leslie population model falls under the category of projection. It is a discrete-time and

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age-structured matrix model for the growth of a closed population, but can be extended to a Leslie-type model which allows for immigration; e.g. [4], [11]. An overview on matrix population models, including Leslie models for populations in time-varying, deterministic or stochastic environments, is provided by Caswell [3]. Rogers [10] focuses on multiregional models. They describe the dynamics of a population which is dispersed over different spatial patches and allow for migration in between. For an ecological system in a multi-patch stochastic environment with different time scales for migration and vital rates, recently Alonso and Sanz [1] showed the application of an aggregation method in order to obtain a reduced stochastic Leslie model.

Our contribution is a new mathematical model for urban and rural populations, which is an extension of the classical Leslie model and allows for migration from rural to urban areas and in the opposite direction. This model is introduced in Section 2. It is able to project the current population structure into the future and permits an analysis of the long-run impact of different vital patterns and migration scenarios on population growth and urbanization. First, a hypothetical population is considered in Section 3, then an application to the case of Turkey is presented in Section 4. Finally, Section 5 gives a summary and some outline for further research. All computations are carried out in R [9].

## 2 The Model

Our model deals with four populations of females: city natives, village natives, and two classes of migrants which are distinguished by destination into city migrants and village migrants. Time proceeds in discrete steps; for illustration purposes we choose 15 years in this and the following section (but 5-year steps in our application to Turkey in Section 4). Accordingly, the four populations are structured by three 15-year intervals of age covering reproductive ages. The age-specific fertility, mortality and migration patterns are assumed to be constant through time. They may be village- or city-specific. In particular, the migrants' vital rates may differ from those of the natives as well, while second generation migrants are assumed to behave like natives with this respect; they are counted as natives actually.

Our model is defined by the relationship and the matrix displayed in Table 1; all symbols are defined in Table 2. Schematically, the matrix  $M_I$  can also be written as

$$M_I = \begin{pmatrix} M_I^{\text{[city native]}} & M_I^{\text{[city migrant]}} & M_I^{\text{[village native]}} & M_I^{\text{[village migrant]}} \\ M_I^{\text{[city native]}} & M_I^{\text{[city migrant]}} & M_I^{\text{[village native]}} & M_I^{\text{[village migrant]}} \\ M_I^{\text{[village native]}} & M_I^{\text{[village migrant]}} & M_I^{\text{[city native]}} & M_I^{\text{[city migrant]}} \\ M_I^{\text{[village native]}} & M_I^{\text{[village migrant]}} & M_I^{\text{[city native]}} & M_I^{\text{[city migrant]}} \end{pmatrix} \quad (1)$$

with sub-matrices  $M_I^{\text{[from]}}$  indicating possible transitions within one time step. Such a sub-matrix will be a zero matrix whenever the corresponding transition is impossible (as in the case  $M_I^{\text{[city native]}}$ ). The element-by-element sum of submatrices along a column of the partitioned  $M_I$  will result in a usual Leslie matrix, which in turn defines population streams channeled to several possible destinations by means of the migration pattern.

**Theorem:** Consider a structured population evolving according to  $N_t = M_I \cdot N_{t-1}$ .

Let the following conditions be satisfied: (i) migration probabilities from city to village and from

village to city are positive for any (not necessarily the same) age class, (ii) survival probabilities are all positive, and (iii) fertility rates are positive for any two adjacent age classes. Then:

- a) There exists a stable population structure  $\tilde{N}$  and a  $\lambda \in \mathbb{R}$  such that

$$\lambda \cdot \tilde{N} = M_I \cdot \tilde{N}. \quad (2)$$

Here,  $\lambda$  is the maximum eigenvalue of  $M_I$ .

- b) The future long-run growth rate of an initial population is given by the maximum eigenvalue of  $M_I$ .
- c) All four population segments (city native, city migrant, village native, village migrant) will ultimately grow with the same rate.

**Proof:** The projection matrix  $M_I$  is a non-negative square matrix, irreducible and primitive. Therefore, classical Perron-Frobenius theory can be applied, see e.g. Seneta [12] and Caswell [3].

Irreducibility and primitivity can be evaluated from the transition diagrams in Figures 1 and 2: The “life cycle graph” is strongly connected, i.e. each pair of nodes is connected in the sense that one node can be reached from the other within a finite number of transitions. The greatest common divisor of loop lengths is 1. This holds if only the conditions of the theorem are satisfied.

Then, according to the Perron-Frobenius theorem, there exists a real and positive eigenvalue  $\lambda$  which dominates any other eigenvalue of  $M_I$ . The eigenvectors associated to  $\lambda$  are strictly positive and unique to constant multiples. In particular, there exists a right eigenvector  $\tilde{N}$  such that equation (2) holds. It follows what is known as the strong ergodic theorem, that  $\lambda$  completely determines the long-term dynamics of the population:

$$\lim_{t \rightarrow \infty} \frac{N_t}{\lambda^t} = c \cdot \tilde{N} \quad (3)$$

In the long run, the population will grow at a rate given by  $\lambda$ , with a stable population structure proportional to  $\tilde{N}$ . In particular, this ultimate growth rate carries over to all four population segments.  $\square$

$$\begin{aligned}
& \left. \begin{array}{l} \text{city native} \\ \text{city migrant} \\ \text{village native} \\ \text{village migrant} \end{array} \right\} \begin{pmatrix} n_{c1t} \\ n_{c2t} \\ n_{c3t} \\ \hline n_{c1t}^* \\ n_{c2t}^* \\ n_{c3t}^* \\ \hline n_{v1t} \\ n_{v2t} \\ n_{v3t} \\ \hline n_{v1t}^* \\ n_{v2t}^* \\ n_{v3t}^* \end{pmatrix} = \begin{pmatrix} f_{c1}\bar{m}_{c1} & f_{c2}\bar{m}_{c2} & f_{c3}\bar{m}_{c3} & f_{c1}\bar{m}_{c1}^* & f_{c2}\bar{m}_{c2}^* & f_{c3}\bar{m}_{c3}^* & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{c1}\bar{m}_{c1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & p_{c2}\bar{m}_{c2} & 0 & 0 & 0 & 0 & f_{v1}\bar{m}_{v1} & f_{v2}\bar{m}_{v2} & f_{v3}\bar{m}_{v3} & f_{v1}\bar{m}_{v1}^* & f_{v2}\bar{m}_{v2}^* & f_{v3}\bar{m}_{v3}^* \\ \hline 0 & 0 & 0 & p_{c1}\bar{m}_{c1}^* & 0 & 0 & p_{v1}\bar{m}_{v1} & 0 & 0 & p_{v1}\bar{m}_{v1}^* & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & p_{c2}\bar{m}_{c2}^* & 0 & 0 & p_{v2}\bar{m}_{v2} & 0 & 0 & p_{v2}\bar{m}_{v2}^* & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & f_{v1}\bar{m}_{v1} & f_{v2}\bar{m}_{v2} & f_{v3}\bar{m}_{v3} & f_{v1}\bar{m}_{v1}^* & f_{v2}\bar{m}_{v2}^* & f_{v3}\bar{m}_{v3}^* \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{v1}\bar{m}_{v1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{v2}\bar{m}_{v2} & 0 & 0 & 0 & 0 \\ \hline f_{c1}m_{c1} & f_{c2}m_{c2} & f_{c3}m_{c3} & f_{c1}m_{c1}^* & f_{c2}m_{c2}^* & f_{c3}m_{c3}^* & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{c1}m_{c1} & 0 & 0 & p_{c1}m_{c1}^* & 0 & 0 & 0 & 0 & 0 & p_{v1}\bar{m}_{v1}^* & 0 & 0 \\ 0 & p_{c2}m_{c2} & 0 & 0 & p_{c2}m_{c2}^* & 0 & 0 & 0 & 0 & 0 & p_{v2}\bar{m}_{v2}^* & 0 \end{pmatrix} \\
& \left. \begin{array}{l} \text{city native} \\ \text{city migrant} \\ \text{village native} \\ \text{village migrant} \end{array} \right\} \begin{pmatrix} n_{c1,t-1} \\ n_{c2,t-1} \\ n_{c3,t-1} \\ \hline n_{c1,t-1}^* \\ n_{c2,t-1}^* \\ n_{c3,t-1}^* \\ \hline n_{v1,t-1} \\ n_{v2,t-1} \\ n_{v3,t-1} \\ \hline n_{v1,t-1}^* \\ n_{v2,t-1}^* \\ n_{v3,t-1}^* \end{pmatrix}
\end{aligned}$$

Table 1: The relation  $N_t = M_I \cdot N_{t-1}$

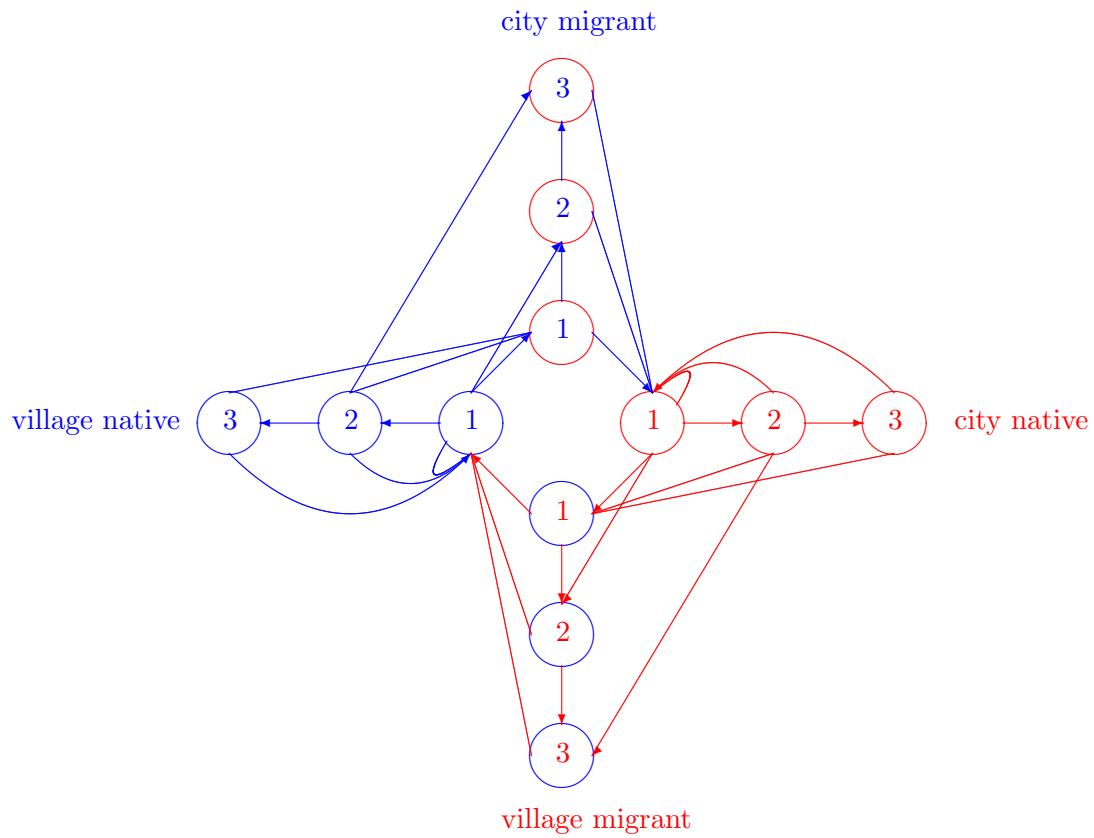


Figure 1: Transitions between states (except migrant to migrant transitions)

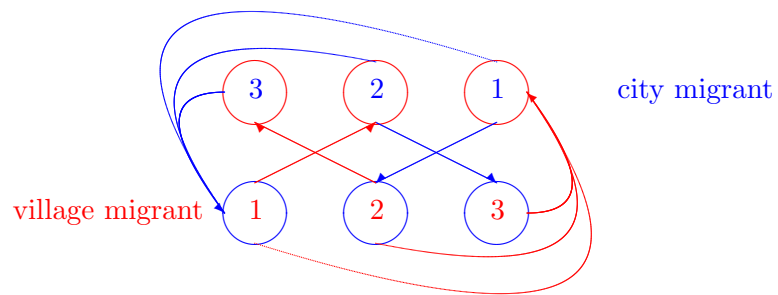


Figure 2: Transitions between migrant states

Symbol	Definition
$f_{ci}$	the average number of girls born to a native of city in age class $i$ , and surviving to the next age class
$f_{vi}$	the average number of girls born to a native of village in age class $i$ , and surviving to the next age class
$f_{ci}^*$	the average number of girls born to a migrant (from village to city) in age class $i$ , and surviving to the next age class
$f_{vi}^*$	the average number of girls born to a migrant (from city to village) in age class $i$ , and surviving to the next age class
$p_{ci}$	the survival probability that a native of city now in age class $i$ , will survive to be in $i + 1$
$p_{vi}$	the survival probability that a native of village now in age class $i$ , will survive to be in $i + 1$
$p_{ci}^*$	the survival probability of a migrant (from village to city) now in age class $i$ , will survive to be in $i + 1$
$p_{vi}^*$	the survival probability of a migrant (from city to village) now in age class $i$ , will survive to be in $i + 1$
$m_{ci}$	probability that a city person (native of city) in age group $i$ , will migrate to village
$\bar{m}_{ci}$	probability that a city person (native of city) in age group $i$ , will NOT migrate to village; $\bar{m}_{ci} = 1 - m_{ci}$
$m_{vi}$	probability that a village person (native of village) in age group $i$ , will migrate to city
$\bar{m}_{vi}$	probability that a village person (native of village) in age group $i$ , will NOT migrate to city; $\bar{m}_{vi} = 1 - m_{vi}$
$m_{ci}^*$	probability that a migrant (from village to city) in age group $i$ , will migrate again (back to village)
$\bar{m}_{ci}^*$	probability that a migrant (from village to city) in age group $i$ , will NOT migrate again (back to village); $\bar{m}_{ci}^* = 1 - m_{ci}^*$
$m_{vi}^*$	probability that a migrant (from city to village) in age group $i$ , will migrate again (back to city)
$\bar{m}_{vi}^*$	probability that a migrant (from city to village) in age group $i$ , will NOT migrate again (back to city); $\bar{m}_{vi}^* = 1 - m_{vi}^*$
$n_{ct}$	age structured ( $i$ ) city population of natives in period $t$
$n_{vt}$	age structured ( $i$ ) village population of natives in period $t$
$n_{ct}^*$	age structured ( $i$ ) population of migrants from village to city in period $t$
$n_{vt}^*$	age structured ( $i$ ) population of migrants from city to village in period $t$

Table 2: Explanation of symbols in the model

### 3 Example

The following hypothetical example is meant to illustrate the dynamics of the model outlined in Section 2, and to demonstrate how the model can contribute to analyzing the composition of a population. The example is based on a population structured by three 15-year age classes:  $[0,15)$ ,  $[15,30)$ , and  $[30,45)$ .

#### 3.1 Assumptions

The first step in constructing our model population is to define two Leslie matrices describing two populations, “city” and “village”, in isolation (with no migration between them). These two matrices are given as

$$M_{\text{city}} = \begin{pmatrix} 0.10 & 0.20 & 0.20 \\ 0.95 & 0 & 0 \\ 0 & 0.90 & 0 \end{pmatrix}, \quad M_{\text{village}} = \begin{pmatrix} 0.30 & 0.90 & 0.70 \\ 0.90 & 0 & 0 \\ 0 & 0.85 & 0 \end{pmatrix}.$$

Their respective maximum eigenvalues are:  $\lambda_{\text{city}} = 0.7086$ ,  $\lambda_{\text{village}} = 1.2699$ . The resulting growth of the stable populations in a 15-year interval is therefore  $-29.14\%$  (city) and  $+26.99\%$  (village), corresponding to annual rates of  $-2.27\%$  and  $+1.61\%$ , respectively. When considered in isolation, the city population is thus shrinking, while the village population is growing (when stability is reached).

The next step is to link the two populations through assumptions concerning (i) migration probabilities of city and village dwellers and migrants, (ii) survival probabilities of migrants, (iii) fertility rates of migrants. The assumptions are:

- Migration probabilities may depend on the origin (city or village), but are equal across age classes, that is:

$$m_{c1} = m_{c2} = m_{c3} = m_c, \quad m_{v1} = m_{v2} = m_{v3} = m_v, \quad (4)$$

and a migrant will stay at her destination and won't migrate back, that is:

$$m_{c1}^* = m_{c2}^* = m_{c3}^* = 0, \quad m_{v1}^* = m_{v2}^* = m_{v3}^* = 0. \quad (5)$$

- Migrants will retain the survival probabilities of their respective origin, that is:

$$p_{ci}^* = p_{vi}, \quad p_{vi}^* = p_{ci}, \quad i = 1, 2, 3. \quad (6)$$

- As far as fertility is concerned, migrants will “split the difference” between city and village fertility, that is, the age-specific fertility rate of migrants is obtained as the arithmetic mean of the respective age-specific fertility rates of city and village population. In symbols:

$$f_{ci}^* = f_{vi}^* = 0.5 \cdot (f_{ci} + f_{vi}), \quad i = 1, 2, 3. \quad (7)$$

This leads us to the matrix  $M_I$  displayed in Table 3. A comparison of the matrices  $M_I$  in Tables 1 and 3 reveals that  $M_I^{\text{[city migrant] [village migrant]}}$  and  $M_I^{\text{[village migrant] [city migrant]}}$  are now zero matrices, reflecting our assumption that migration back is excluded ( $m_c^* = m_v^* = 0$ ). Considering the growth properties of city (shrinking) and village (growing) populations, the theorem in Section 2 implies that there is a migration pattern, expressed by  $m_c$  and  $m_v$ , that will ultimately lead to a *stationary* total population. We shall first present an example of the transient behaviour of the total population before we come back to the steady-state properties of  $M_I$ .

$0.10\bar{m}_c$	$0.20\bar{m}_c$	$0.20\bar{m}_c$	0.20	0.55	0.45	0	0	0	0	0	0
$0.95\bar{m}_c$	0	0	0	0	0	0	0	0	0	0	0
0	$0.90\bar{m}_c$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$0.30m_v$	$0.90m_v$	$0.70m_v$	0	0	0
0	0	0	0.90	0	0	$0.90m_v$	0	0	0	0	0
0	0	0	0	0.85	0	0	$0.85m_v$	0	0	0	0
0	0	0	0	0	0	$0.30\bar{m}_v$	$0.90\bar{m}_v$	$0.70\bar{m}_v$	0.20	0.55	0.45
0	0	0	0	0	0	$0.90\bar{m}_v$	0	0	0	0	0
0	0	0	0	0	0	0	$0.85\bar{m}_v$	0	0	0	0
$0.10m_c$	$0.20m_c$	$0.20m_c$	0	0	0	0	0	0	0	0	0
$0.95m_c$	0	0	0	0	0	0	0	0	0.95	0	0
0	$0.90m_c$	0	0	0	0	0	0	0	0	0.90	0

Table 3: Matrix  $M_I$ , resulting from the assumptions in the example

### 3.2 Movement Towards Stability

With initial population

$$N'_1 = (\underbrace{1000, 1000, 1000}_{\text{city native}}, \underbrace{0, 0, 0}_{\text{city migrant}}, \underbrace{1000, 1000, 1000}_{\text{village native}}, \underbrace{0, 0, 0}_{\text{village migrant}}), \quad (8)$$

and with migration probabilities  $m_c = 0.1$  (from city to village) and  $m_v = 0.3$  (from village to city) Figure 3 shows a plot of the 12 series given by

$$N_t = M_I \cdot N_{t-1}, \quad t = 2, 3, \dots, 25, \quad (9)$$

that is, the evolution of the composition of this population. The maximum eigenvalue of  $M_I$ , as displayed in Table 3, is  $\lambda = 0.9920$ , so that the population shrinks in the long run: The low fertility of the city population prevails due to the high village-to-city migration ( $m_v = 0.3$ ). Lowering  $m_v$  somewhat (to  $m_v = 0.28$ , say) would make the population grow. For the impact of migration probabilities on the long-run behaviour of the population, see also Figure 4.

### 3.3 The Impact of Migration on Long-Run Growth and Urbanization

The long-run impact of migration probabilities on growth and urbanization can now be studied via the matrix  $M_I$  in Table 3 by letting the growth factor (that is, the maximum eigenvalue) and long-run urbanization be a function of  $(m_v, m_c)$ . Urbanization is computed as the share of population in the first six components of the population vector. (Technically, urbanization is the sum of the first six components of the eigenvector belonging to the maximum eigenvalue, divided by the total sum.) Plots of resulting contour lines are displayed in Figure 4. In particular, a higher city-to-village migration can offset a higher village-to-city migration in terms of equal long-run growth, where the ratio depends on the location of  $(m_v, m_c)$ . Similar remarks apply to long-run urbanization.

## 4 The Case of Turkey

The goal of this section is to apply the model of Section 2 to the population of Turkey, as characterized by recent data concerning age structure, fertility, survival, and migration. We

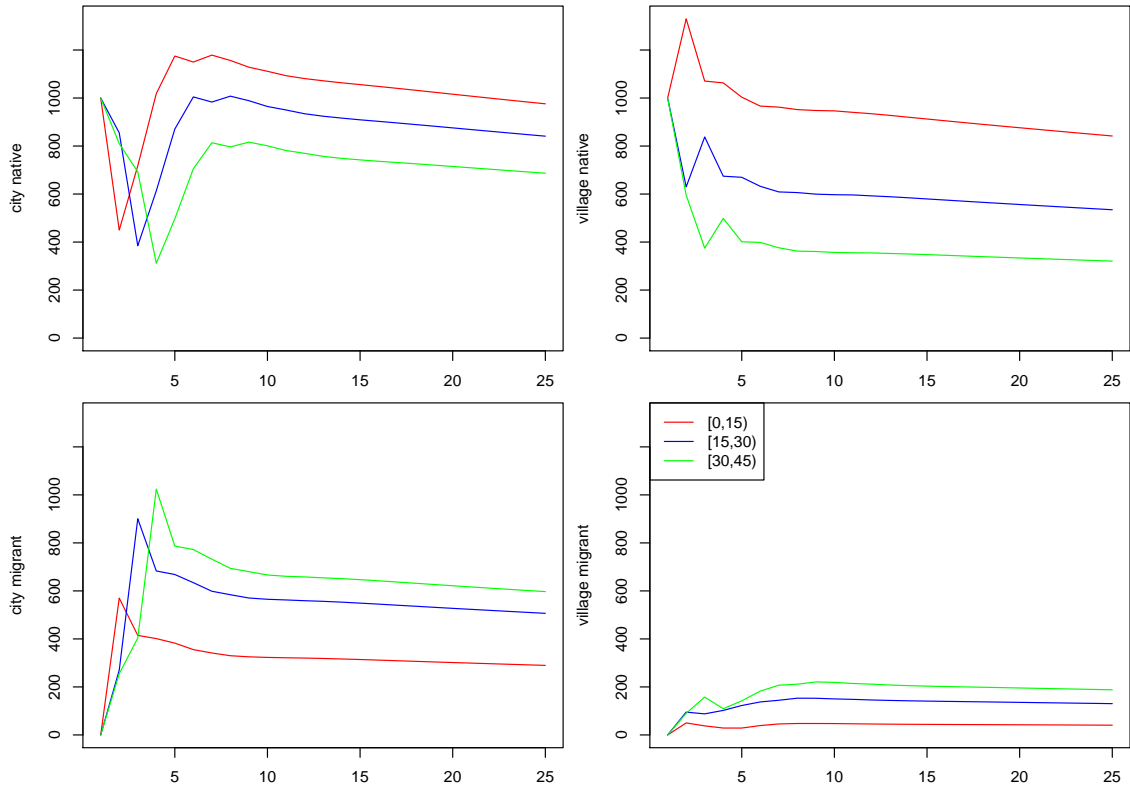


Figure 3: Evolution of initial population

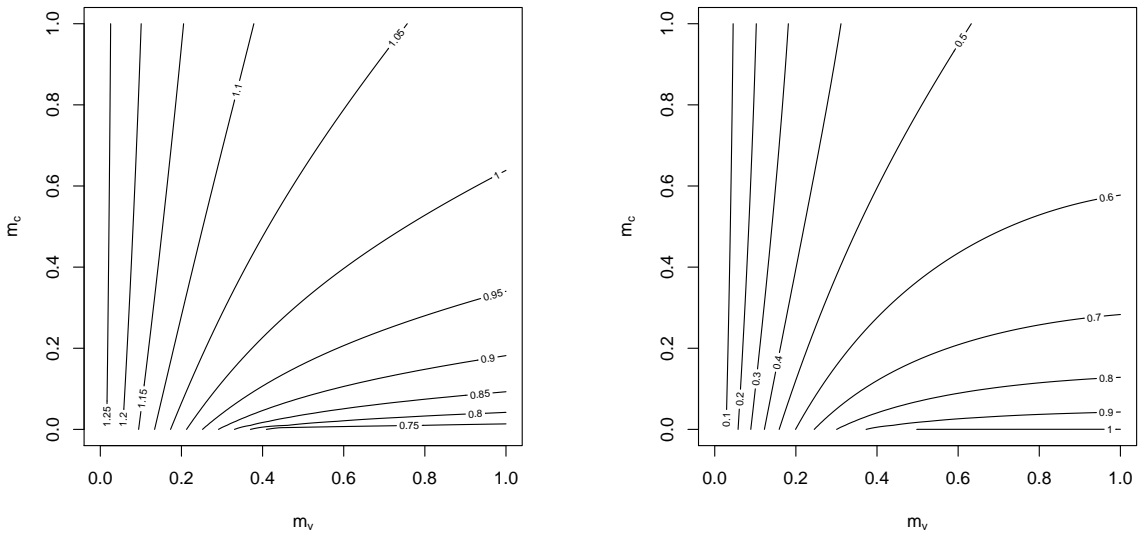


Figure 4: Impact of migration on growth (left) and urbanization (right)

shall first give an account of the data and explain assumptions made to obtain the projection matrix  $M_I$ . This will enable us to compare the actual situation in Turkey with its stable counterpart, based on the model developed in Section 2, and to discuss the impact of fertility and migration levels on long-run growth and urbanization, similar to the procedures outlined in Section 3.

#### 4.1 Turkish Population Data

All data to be used in the definition of  $M_I$  are reported in Table 4.

	age group									
	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
fertility rates										
city	0.0000	0.0000	0.0000	0.1100	0.3150	0.3150	0.1780	0.0830	0.0280	0.0000
village	0.0000	0.0000	0.0000	0.1180	0.4030	0.3950	0.2350	0.1200	0.0400	0.0150
survival probabilities										
	0.9574	0.9969	0.9979	0.9971	0.9962	0.9953	0.9940	0.9918	0.9883	0.9824
migration probabilities										
to city	0.0500	0.0430	0.0455	0.0631	0.1020	0.0821	0.0531	0.0415	0.0328	0.0264
to village	0.0300	0.0284	0.0265	0.0335	0.0445	0.0402	0.0320	0.0272	0.0261	0.0274

Table 4: Turkish (female) population data used for the projection matrix  $M_I$

Age-specific fertility rates of the urban and rural population in Turkey for the year 2003 are published by the Hacettepe University Institute of Population Studies [6]. — Survival probabilities (year 2000) are taken from WHO [15], abridging the first two intervals. — Substitutes for migration probabilities were obtained as the number of female migrants (village to city or city to village), divided by the total female population in that age group (again village or city, respectively). Both series are published by the Turkish Statistical Institute [13].

Since city- and village-specific survival probabilities were not available, we assume that urban and rural populations have the same survival probabilities. Furthermore, we assume that migrants immediately adopt the destination's (and the natives') migration probabilities, that is,  $m_{ci}^* = m_{ci}$ ,  $m_{vi}^* = m_{vi}$ . As to fertility rates, we repeat the assumption made in Section 3: Migrants' age-specific fertility rates are obtained as the arithmetic means of natives of both locations.

Finally, the initial (female) population  $N_1$  of Turkey (for the year 2000; see [13]) is given as shown in Table 5.

#### 4.2 Actual and Stable Population

The model outlined in Section 2 can now be applied to the population of Turkey, thus permitting a comparison of actual and stable population measures. As in the example above, we focus on urbanization and growth. It is important to keep in mind that all measures refer to the female population aged 0 to 49.

Partitioning the projection matrix  $M_I$  according to (1) reveals the characteristics of city and village population if there were no migration. (This is similar to the starting point in Section 3.)

age class	city native	city migrant	village native	village migrant
[0, 5)	1980044	60404	1208088	59401
[5, 10)	2052625	52389	1218246	58228
[10, 15)	2090782	55365	1217217	55428
[15, 20)	2254619	79729	1263638	75480
[20, 25)	2204497	108059	1058935	98019
[25, 30)	2000720	75409	918105	80349
[30, 35)	1693854	40569	763431	54190
[35, 40)	1652431	31049	748377	44951
[40, 45)	1356984	20614	628241	35412
[45, 50)	1097967	14793	560045	30045

Table 5: The initial (female) population of Turkey (year 2000)

	actual	stable
urbanization	65.0%	58.7%
growth, city, 5-year period	4.552%	1.521%
growth, village, 5-year period	6.350%	1.521%
total growth, 5-year period	5.181%	1.521%
growth, city, annual	0.894%	0.302%
growth, village, annual	1.239%	0.302%
total growth, annual	1.015%	0.302%

Table 6: Urbanization, growth of (female) Turkish population: actual and stable

Projection matrices for the populations evolving in isolation are obtained as

$$M_{I,\text{city}} = M_I^{\text{[city native]}} + M_I^{\text{[city native] [village migrant]}}, \quad M_{I,\text{village}} = M_I^{\text{[village native]}} + M_I^{\text{[village native] [city migrant]}}. \quad (10)$$

Their respective maximum eigenvalues are  $\lambda_{\text{city}} = 0.9954$  and  $\lambda_{\text{village}} = 1.0385$ . This means: Ceteris paribus (i.e. fertility and mortality unchanged), any migration pattern in Turkey will, in the long run, produce growth rates (for 5-year periods) between  $-0.46\%$  and  $+3.85\%$ .

The histograms in Figure 5 compare the actual and stable populations of Turkey. City and village populations were obtained by adding native and migrant populations. The histograms show that the actual population is younger than its stable counterpart. This accounts for the higher actual growth rates when compared to stable growth rates, which leads us to a comparison of actual and stable population measures for the Turkish population. The relevant measures are given in Table 6.

Growth measures referring to the actual population are simply obtained by relating the initial population  $N_1$  to  $N_2 = M_I \cdot N_1$ . Measures for the stable population can be obtained by substituting the right eigenvector belonging to the maximum eigenvalue of  $M_I$  as population vector, and keeping in mind that city and village share the same growth rate.

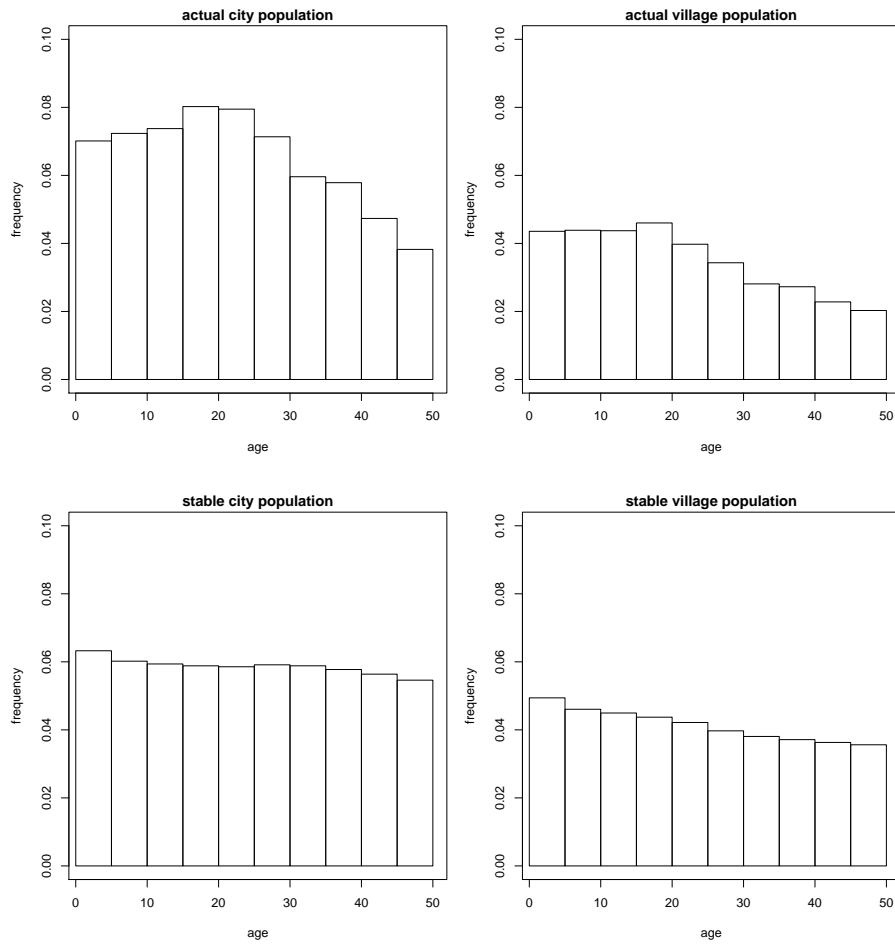


Figure 5: Actual (top) and stable (bottom) female population of Turkey

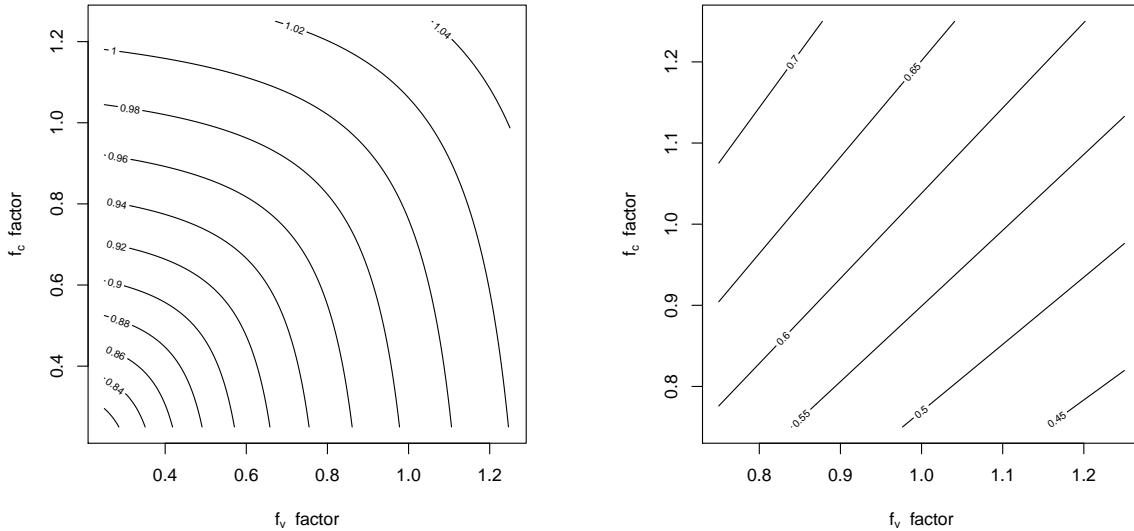


Figure 6: Impact of fertility levels on growth (left) and urbanization (right)

### 4.3 The Impact of Fertility Levels on Growth and Urbanization

In what way do long-run growth and urbanization of the Turkish population depend on urban and rural fertility levels? One procedure to elucidate this hypothetical question is to multiply each fertility rate  $f_{vi}$  (and  $f_{ci}$ ), as given in Table 4, with a common factor, designated as “ $f_v$  factor” (“ $f_c$  factor”, respectively) in the abscissa (ordinate, respectively) of the plots in Figure 6. This operation will also affect migrants’ fertility rates, which are again defined as arithmetic means.

A substitution effect between fertility levels with respect to population growth can be observed (the plot on the left side of Figure 6). Substitutions between  $f_v$  and  $f_c$  will have a strong effect on urbanization, according to the right-hand plot of Figure 6.

### 4.4 The Impact of Migration on Growth and Urbanization

A related question is: How do migration probabilities affect the long-run growth and urbanization? This is displayed in the plots in Figure 7, assuming that migration probabilities do not depend on the age class. (This scenario is analogous to the set-up in Section 3.) As remarked before, any growth factor is between 0.9954 and 1.0385. The impact of the balance between  $m_v$  and  $m_c$  on urbanization can be grave when both probabilities are small, according to the right-hand plot in Figure 7.

## 5 Conclusions and Further Research

We have introduced a new projection-matrix based population model which takes rural-urban and urban-rural migration explicitly into consideration. Perron-Frobenius theory provides insight into the stable behaviour of this model. We have thus developed a framework for the analysis of the impact of fertility rates, survival probabilities, as well as migration patterns on long-run characteristics of a population.

One advantage of the model is that it can be applied to real-world situations using available data. We compile relevant data and undertake an effort to quantify and discuss characteristics

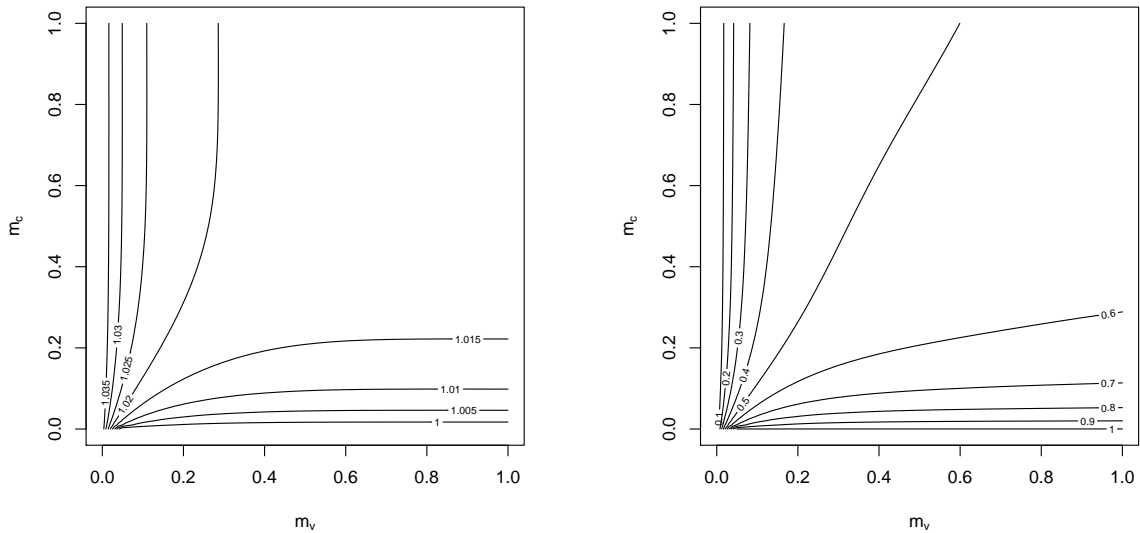


Figure 7: Impact of migration on growth (left) and urbanization (right)

of the population of Turkey. We found that the actual current population of Turkey is younger than its stable counterpart, leading to elevated growth rates. More importantly in our context, we found that, when considered in isolation (with migration computationally removed from population dynamics), urban population is slightly below replacement, while rural is above, resulting — by means of the prevailing migration pattern — in a growing stable population. We compare characteristics of the actual Turkish population with its stable counterpart and find that long-run urbanization will slightly decrease under current conditions, however, on the other hand, slight changes in migration probabilities can have a profound impact on long-run urbanization.

Our approach provides ample opportunity for further research. Two research directions are: (i) to elaborate further characteristics of actual populations, and assess the impact of public policies on the future population structure and its economic consequences, such as the age dependency ratio; (ii) to gain further theoretical insight into the model, for example, the transformation of the model into a Markov chain (this was found useful for other Leslie-type models; see, for example, Schmidbauer and Rösch [11]), which would create a platform for the introduction of reproductive values in order to study migration and fertility from a novel perspective.

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