FM 431: Econometrics of Financial Markets

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- The slides show only a part of what you will learn in this course.

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- The slides were produced using LaTeX and R (the R project; www.R-project.org) on a Linux system.

- Course material is available at www.hs-stat.com/courses/FM431.

Chapter 5:

Models With Trend

5.1 Trends

Series with trends; two examples.

- Key feature of a trend: permanent, non-decaying effect.
- Let's consider two models with trend.
- Each of the two processes results from
  - an assumption about the increment $\Delta y_t = y_t - y_{t-1}$,
  - an assumption about a stochastic perturbation.
- They have a similar structure, but completely different behaviour.

5.1 Trends

Example 1: A model with stationary trend.

- Assumption about increment:

$$\Delta y_t = a_0$$

- The resulting process is perturbed by a stationary component:

$$y_t = y_0 + a_0 t + \alpha(L)\epsilon_t, \quad (\epsilon_t): \text{ white noise}$$

- This is called a trend-stationary model.

5.1 Trends

Example 2: A model with stochastic trend.

- Assumption about increment:

$$\Delta y_t = a_0 + \epsilon_t, \quad (\epsilon_t): \text{ white noise}$$

- The resulting process is:

$$y_t = y_0 + a_0 t + \sum_{i=1}^{t} \epsilon_i$$

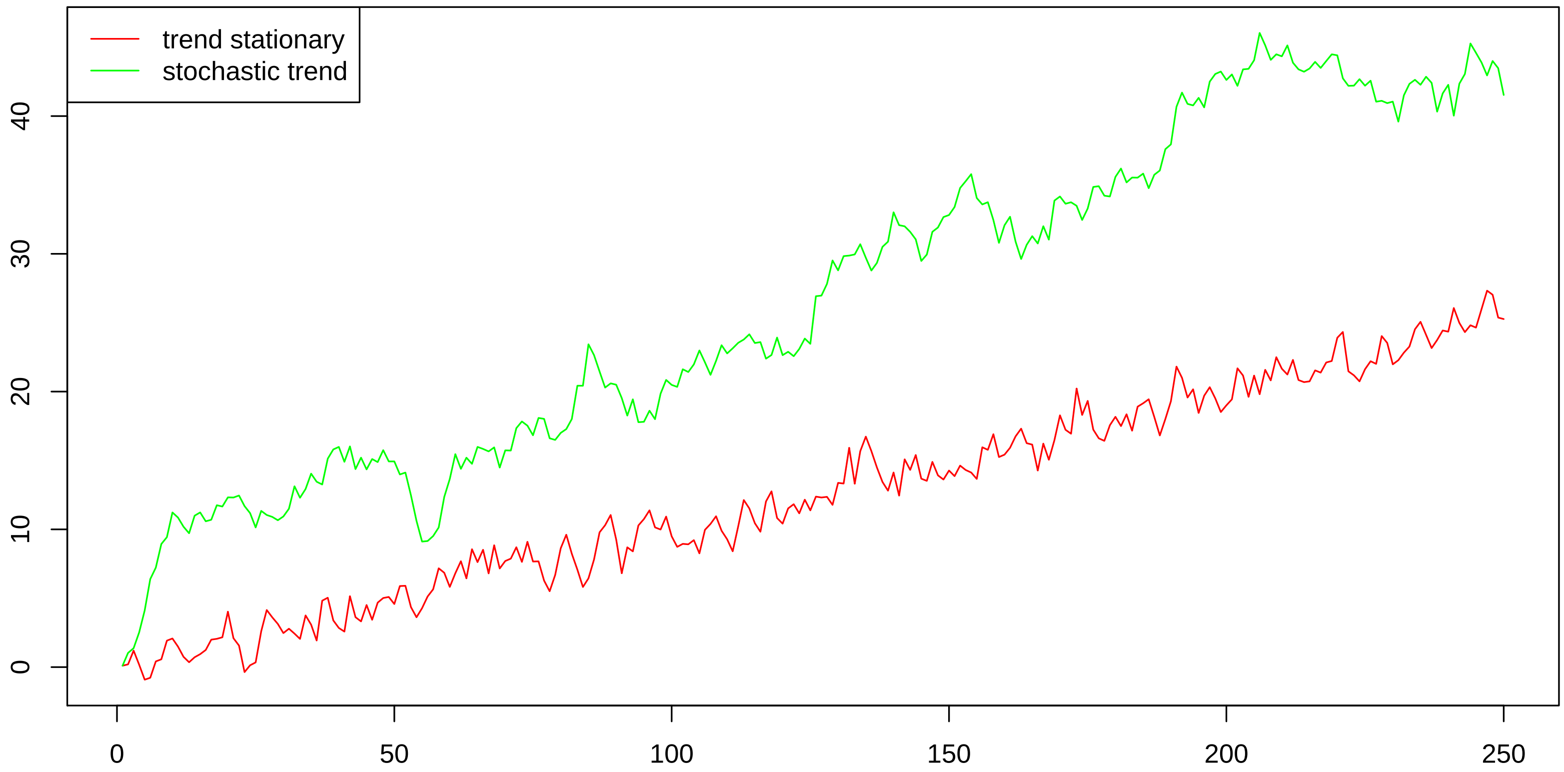
- This is called a model with stochastic trend (or a random walk with drift).

5.1 Trends

- Both models have a term "$a_0 t$".
- Model 2 is perturbed by a random walk.
- We shall now:
  - see simulations of both models,
  - investigate the random walk properties,
  - point out some difficulties with identifying a random walk.

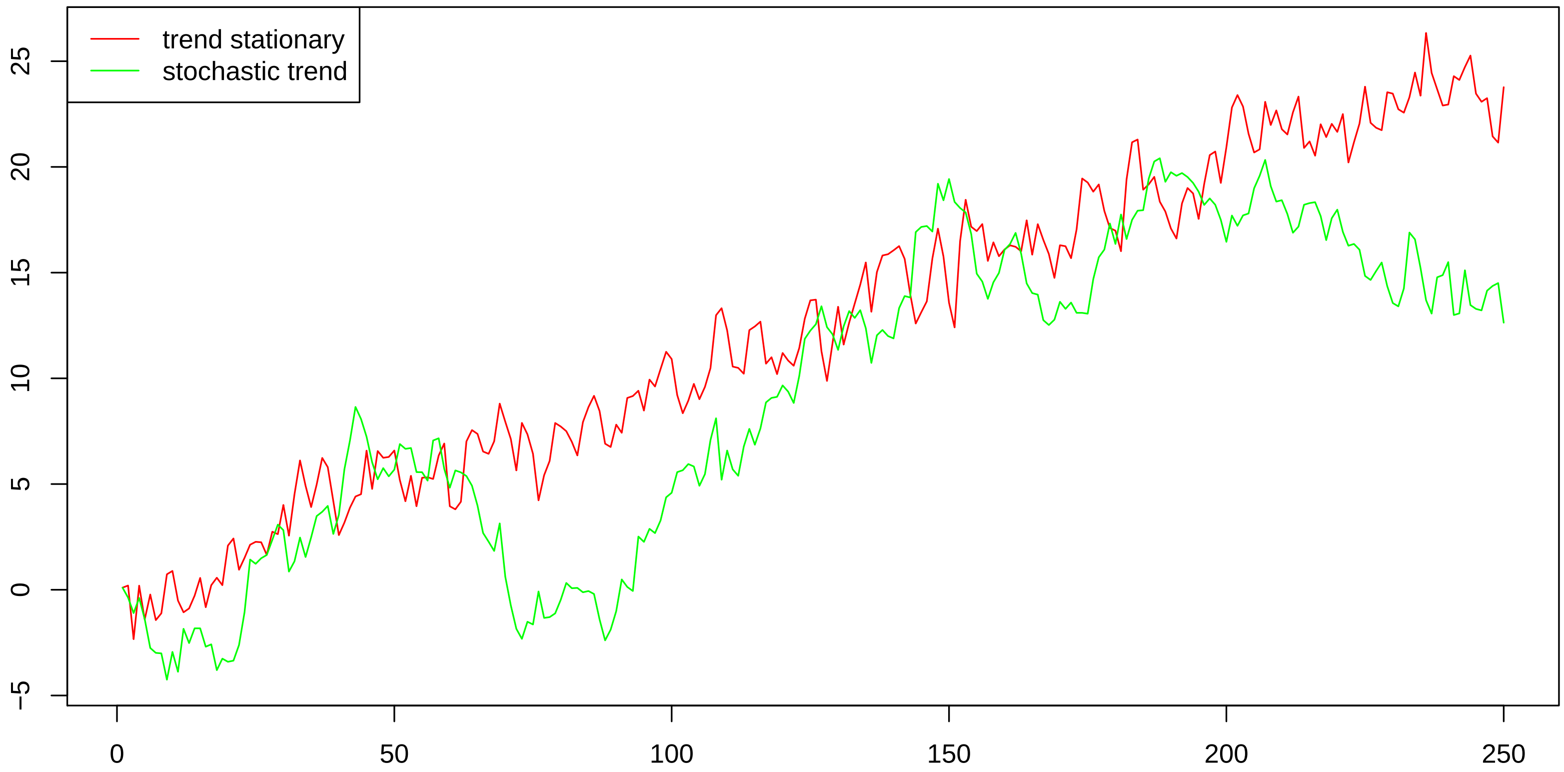
5.1 Trends

A simulation of models 1 and 2.



5.1 Trends

A simulation of models 1 and 2.



5.1 Trends

Properties of a random walk — moments.

- Model:

$$y_t = y_0 + \sum_{i=1}^{t} \epsilon_i, \quad (\epsilon_t): \text{ white noise with } \mathrm{var}(\epsilon_t) = \sigma_\epsilon$$

- Unconditional moments:

$$\mathrm{E}(y_t) = y_0, \quad \mathrm{var}(y_t) = t\sigma_\epsilon^2$$

- Conditional moments:

$$\mathrm{E}(y_{t+1}|y_t) = y_t, \quad \mathrm{var}(y_{t+1}|y_t) = \sigma_\epsilon^2$$

5.1 Trends

Properties of a random walk — autocorrelation.

- Covariance:

$$\mathsf{E}\left[(y_{t+s} - y_0)(y_t - y_0)\right] = \mathsf{E}\left(\sum_{i=1}^{t+s} \epsilon_i \cdot \sum_{i=1}^{t} \epsilon_i\right) = t\sigma_\epsilon^2$$

- Correlation:

$$\rho_s = \mathrm{cor}(y_{t+s}, y_t) = \frac{t\sigma_\epsilon^2}{\sqrt{(t+s)\sigma_\epsilon^2} \cdot \sqrt{t\sigma_\epsilon^2}} = \sqrt{\frac{t}{t+s}}$$

5.1 Trends

Autocorrelation of a random walk.

Conclusions.

- $s \mapsto \rho(s)$ is decreasing; similar to the acf of an AR(1) process.
- The speed of decrease will depend on t.
- The acf is not suitable to distinguish a random walk from an AR(1) process.

5.1 Trends

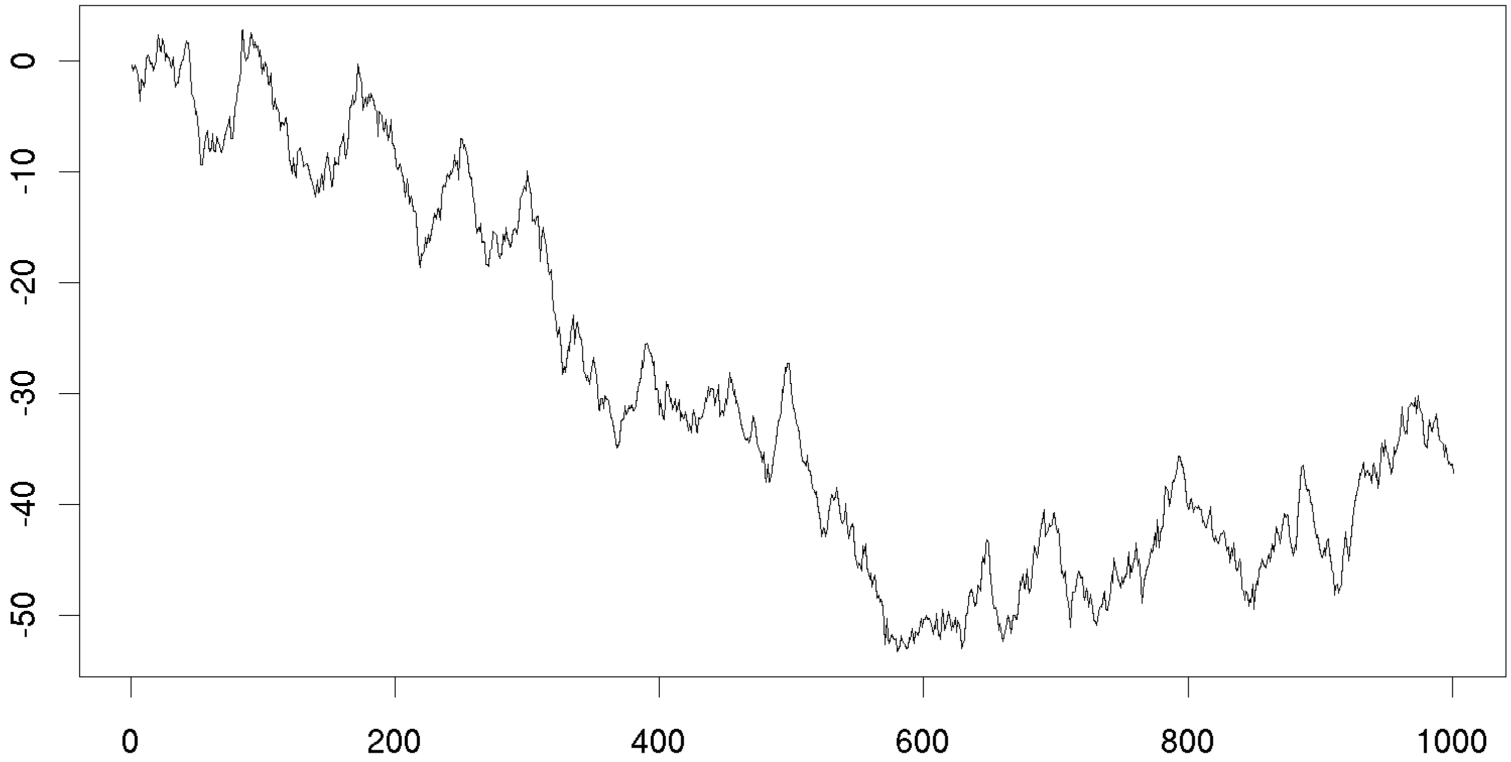
Simulation of a random walk.

We shall now:

- Simulate a (Gaussian) random walk (y_t).
- Draw, such that the dependence on t becomes clear:
  - scatterplots of y_t vs. y_{t-1},
  - the acf of the simulated series.

5.1 Trends

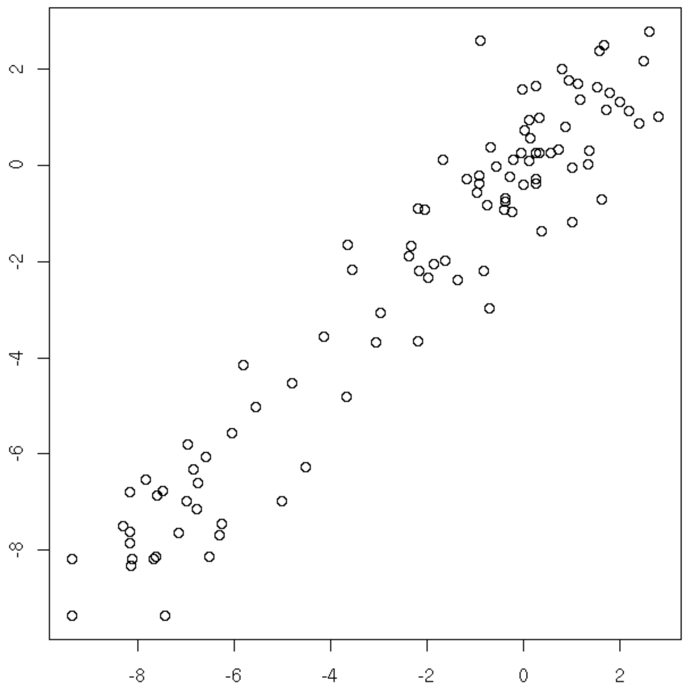
A simulated random walk.



5.1 Trends

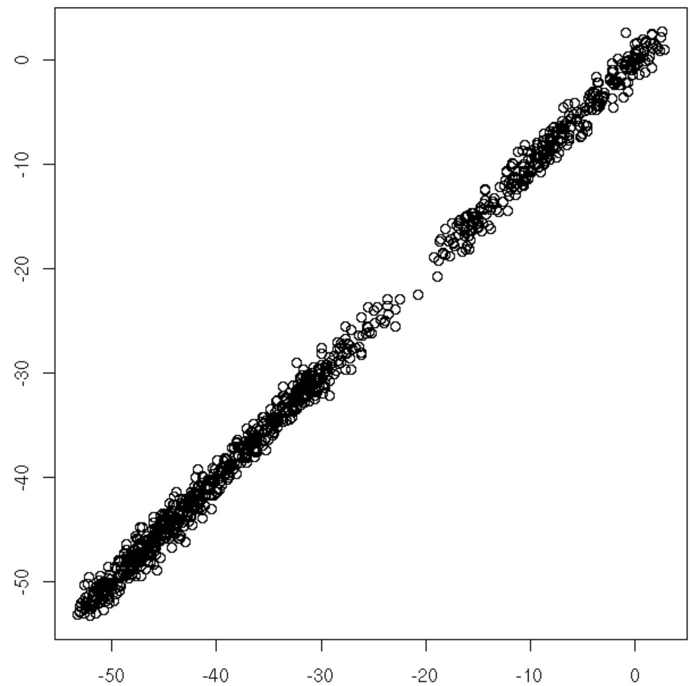
The simulated random walk — scatterplot of y_t vs. y_{t-1}.

First 100 realizations:



correlation: 0.9556

All 1000 realizations:

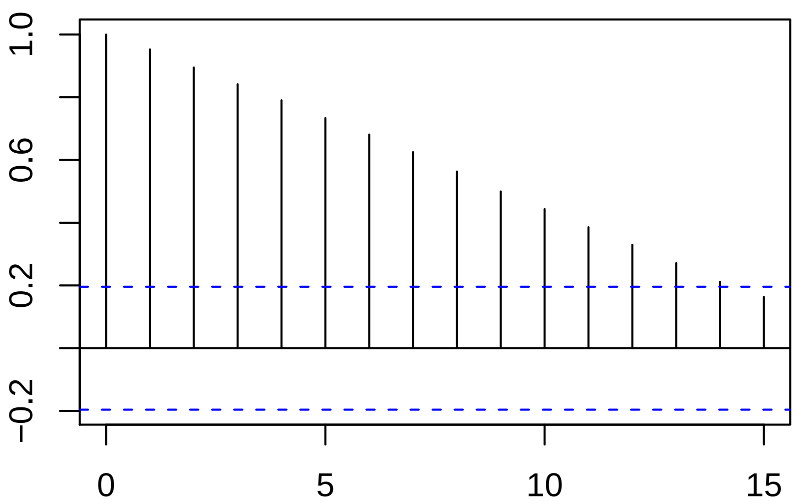


correlation: 0.9982

5.1 Trends

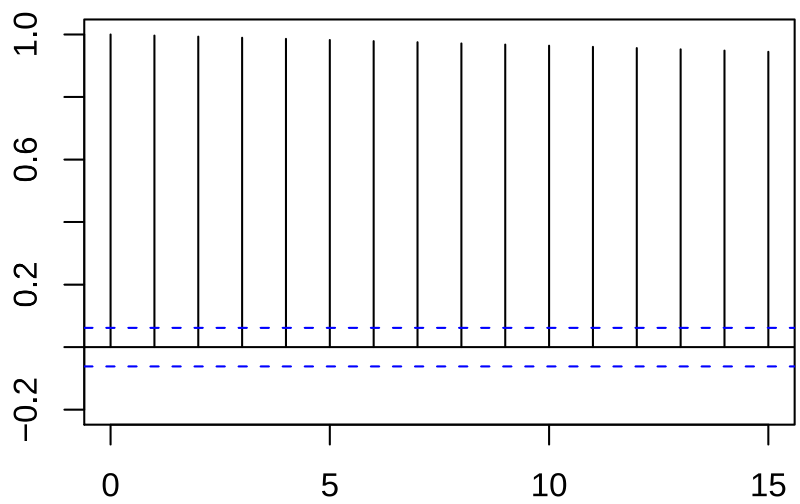
The simulated random walk — autocorrelation function.

First 100 realizations:



autocorrelation (lag 1): 0.9556

All 1000 realizations:



autocorrelation (lag 1): 0.9982

5.1 Trends

Statistical inference.

- Consider the model $y_t = y_0 + a y_{t-1} + \epsilon_t$.
- If $a < 1$, the correlation of y_t and y_{t-1} estimates a.
- If $a = 1$, the correlation of y_t and y_{t-1} underestimates a.
- This is why special tests are needed to test

$$H_0 : a = 1 \ \text{ against } \ H_1 : a < 1.$$

5.1 Trends

News impact.

- One reason why it is important to distinguish between random walk and AR(1): News impact is different.
- AR(1): $y_t = a_0 + a_1 y_{t-1} + \epsilon_t$. Then:

$$y_t = \frac{a_0}{1 - a_1} + \sum_{i=0}^{\infty} a_1^i \epsilon_{t-i}, \quad \text{so: } \frac{\partial y_{t+s}}{\partial \epsilon_t} = a_1^s.$$

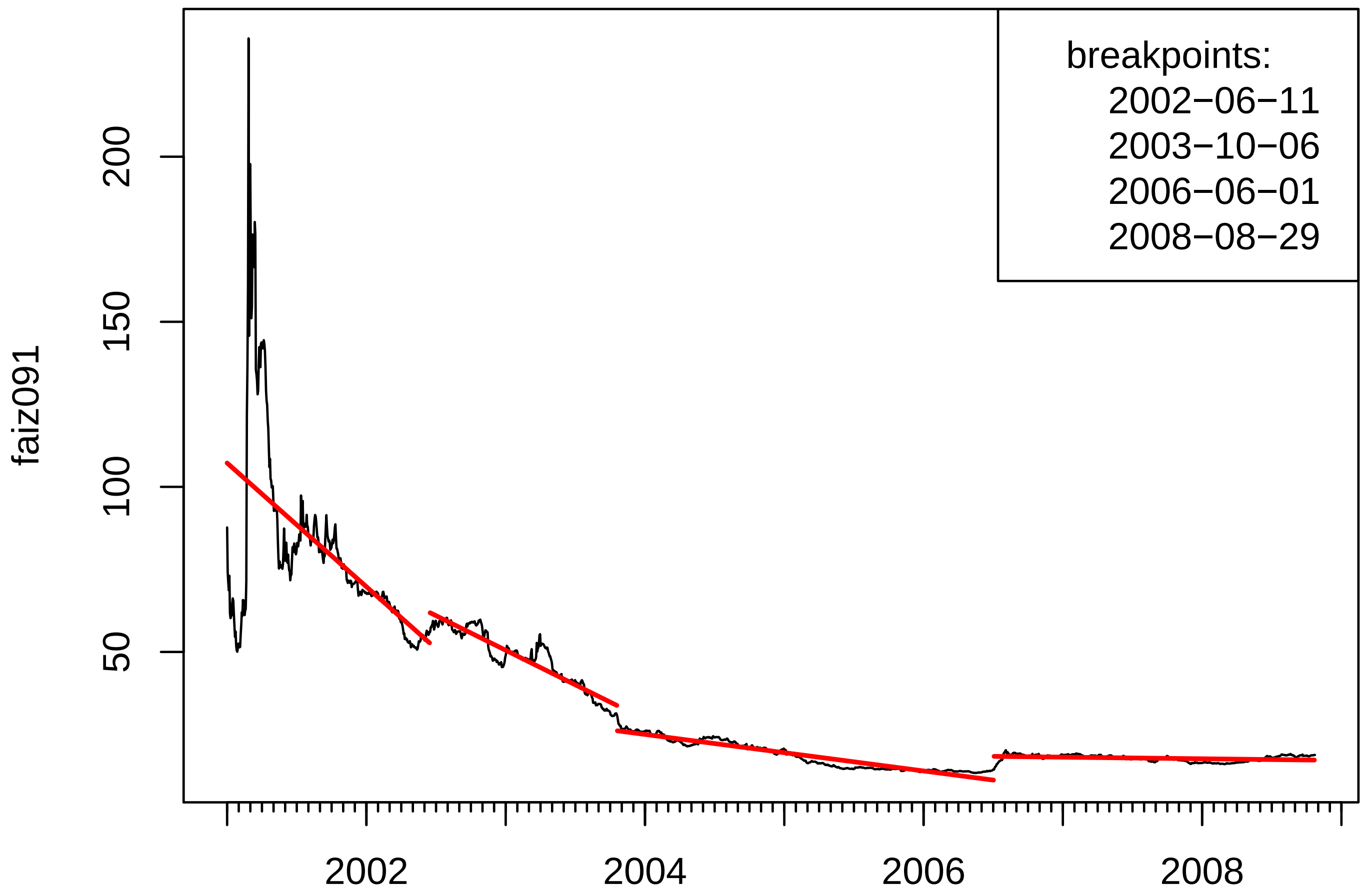
- Random walk: $y_t = y_{t-1} + \epsilon_t$. Then:

$$y_t = y_0 + \sum_{i=1}^{t} \epsilon_i, \quad \text{so: } \frac{\partial y_{t+s}}{\partial \epsilon_t} = 1.$$

  Permanent, but random, change in mean, justifies the term "stochastic trend".

Some Figures

Interest Rate in Turkey.



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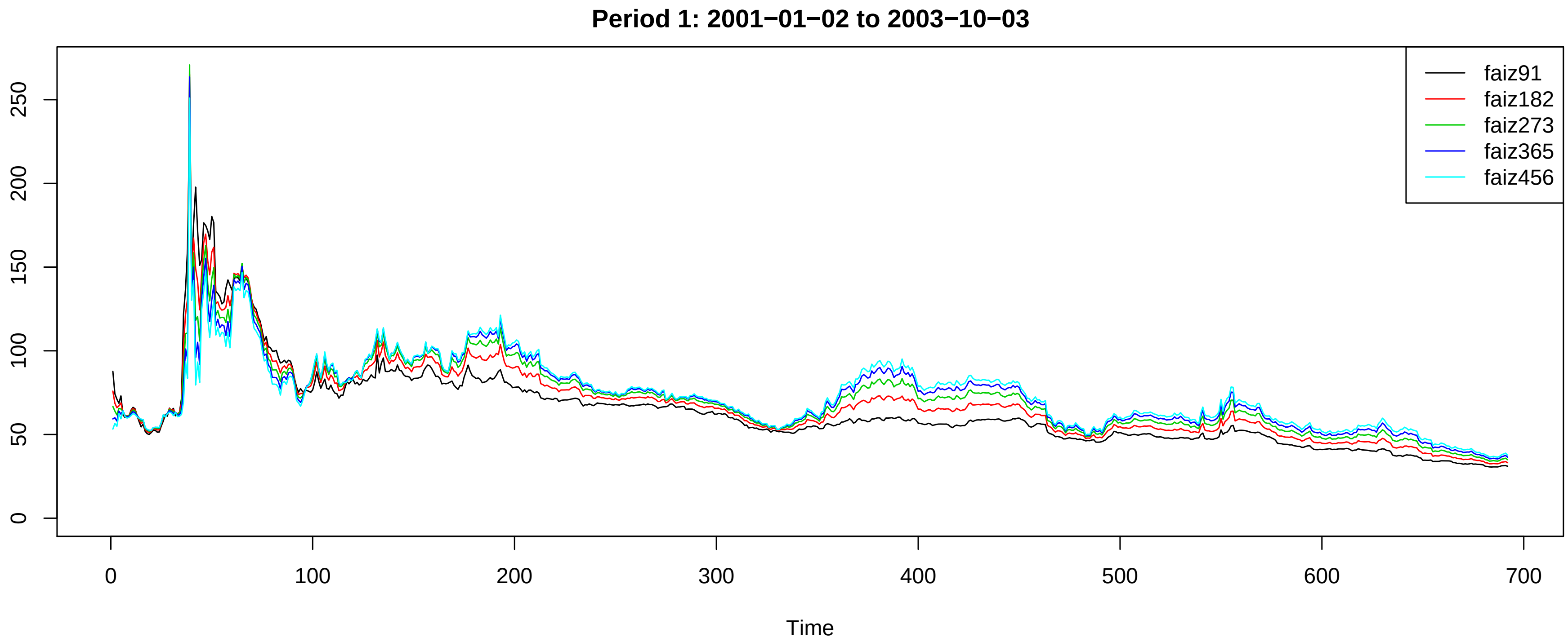

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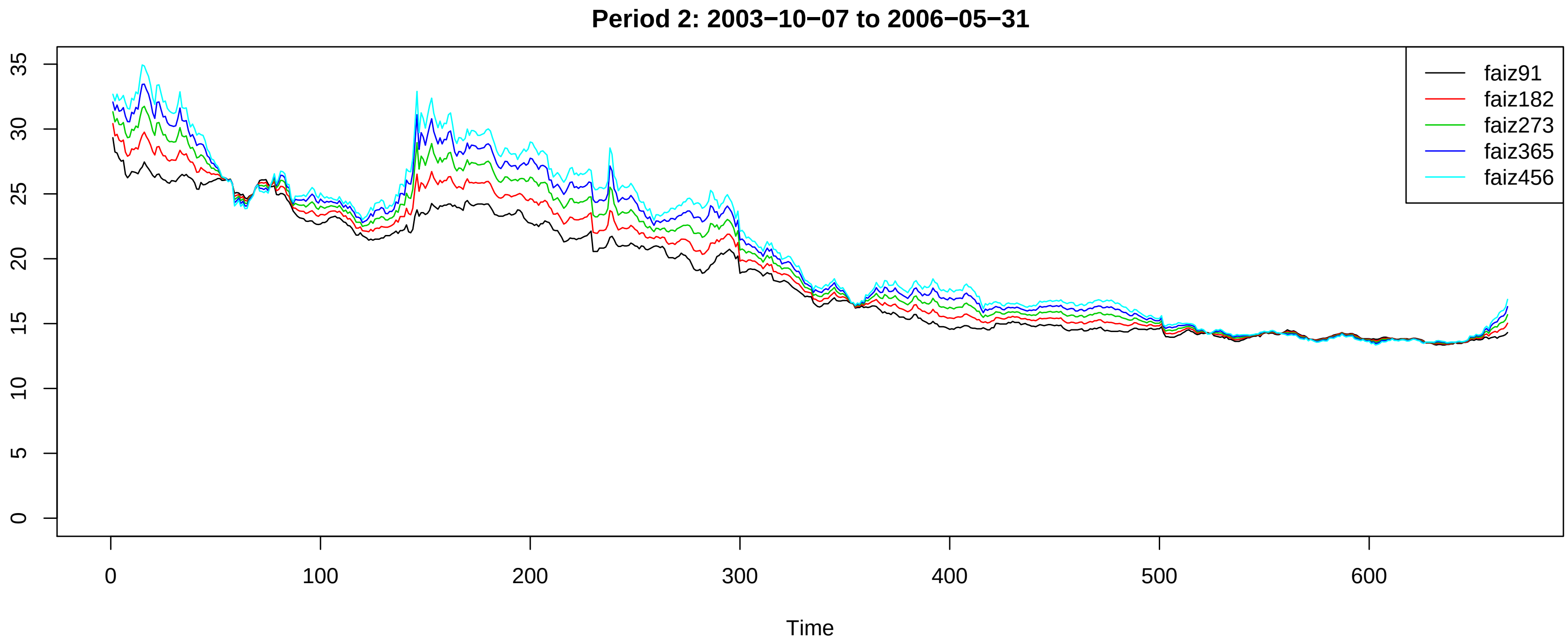

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