FM 431: Econometrics of Financial Markets

Harald Schmidbauer

İSTANBUL BİLGİ ÜNİVERSİTESİ

© Harald Schmidbauer & Angi Rösch, 2008

About These Slides

- The present slides are not self-contained; they need to be explained and discussed.
- The slides show only a part of what you will learn in this course.
- Even though being a "work in progress" and subject to revision, the slides constitute copyrighted material.
  If you want to reproduce or copy anything from the slides, please ask:

Harald Schmidbauer	**harald** at **hs-stat** dot **com**
Angi Rösch	**angi.r** at **t-online** dot **de**

- The slides were produced using LaTeX and R (the R project; www.R-project.org) on a Linux system.
- Course material is available at www.hs-stat.com/courses/FM431.

Chapter 2:

Are Asset Returns Predictable?

2.1 Introduction

Predictability.

- Predictability presupposes some structure in a time series.
- Forecasting program:
  - Use a suitable method to reveal the structure.
  - Use past observations to predict the future.
- This will work only if the past contains information about the future.

2.1 Introduction

Examples.

- Monthly car sales in Turkey:
  - Suppose this month very many cars were sold, due to a tax increase expected for next month.
  - Forecast for next month: Car sales will go down.
    (Sales were anticipated!)

- Monthly tourist arrivals to Turkey:
  - Suppose a terrorist attack has scared off tourists.
  - We can try to predict when things "normalize" again.

2.1 Introduction

Financial markets.

- However, efficient financial markets work in a different way.
- Suppose there was a method that predicts a price increase of a certain stock tomorrow. What would you (actually, all of us) do?
  - Buy the stock today, to sell it tomorrow!
  - Increased demand will drive up the price right now, not tomorrow.
  - Collective selling tomorrow will drive profits to zero. . .

2.1 Introduction

The efficient-market hypothesis.

- It says: Asset prices reflect all available information.
  (Which information? — See below!)

- So: Financial markets process all available information immediately — they are “informationally efficient”.

- It is then not possible to use past information to forecast the future. (All past information is already included in today’s asset prices. No information surplus bearing on the future.)

- Information, news: appears randomly in the future!

2.1 Introduction

The random walk hypothesis.

- In a certain sense, this is in line with the efficient-market hypothesis.
- It states that stock market prices evolve according to a random walk.
- Then: We cannot forecast prices, using past information.
- Reminder: One random walk model is $(X_t)_{t\geq 0}$ with

$$X_t = X_{t-1} + \epsilon_t, \quad X_0 = 0, \quad \epsilon_t \sim \mathsf{N}(0, \sigma^2) \text{ iid.}$$

2.1 Introduction

Outlook. We shall now:

- Introduce some simple random walk models to formulate hypotheses:
  - RWH I: Asset returns are iid.
  - RWH II: Asset returns are independent.
  - RWH III: Asset returns are uncorrelated.
- Show how to test each hypothesis.
- Carry out the test for some examples.

We could say: A stochastic model serves as a "looking glass" to find traces of predictability in an asset price series.

2.1 Introduction

Reminder: Prices and their changes.

- Let

  P_t = closing price of a stock on day (week/month) t.

- A return for day (week/month) t can be defined as

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \qquad \text{or} \qquad r_t = \ln \frac{P_t}{P_{t-1}}.$$

- Using elementary calculus, it can be shown that $R_t \approx r_t$, provided that $P_t \approx P_{t-1}$.

2.2 The Random Walk Hypothesis I

A random walk with iid asset price changes.

- The RWH I states that prices evolve according to:

$$\ln P_t = \mu + \ln P_{t-1} + \epsilon_t, \quad (\epsilon_t): \text{ iid}$$

or

$$P_t = P_{t-1} \cdot \mathrm{e}^{\mu} \cdot \mathrm{e}^{\epsilon_t}$$

- Discrete version of exponential Brownian motion!

- For returns:

$$r_t = \mu + \epsilon_t, \quad (\epsilon_t): \text{ iid}$$

2.2 The Random Walk Hypothesis I

Behaviour of the sign sequence.

- The RWH I implies that the *sign of returns* sequence should behave like a sequence of iid Bernoulli experiments.
- We can thus test the RWH I for a given sequence of returns.
- A powerful tool: iterations in the sequence.
- Compare these three sequences:

  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
  0 1 1 1 1 0 0 0 0 1 0 0 0 1 1 1 1 0 1 0 1 0 0 1 1 0 0 0 0 0 1 1 1 1 1 0

2.2 The Random Walk Hypothesis I

Iterations.

Definition: The sequence $(x_{j+1}, \ldots, x_{j+l-1}, x_{j+l})$ is called an *iteration* if

$$x_j \neq x_{j+1} = \ldots = x_{j+l-1} = x_{j+l} \neq x_{j+l+1}.$$

The number l is called the *length of the iteration.*

Example: The sequence $1, 1, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 1, 0$ has eight iterations:

$$\underbrace{1 \quad 1}_{l=2} \quad \underbrace{0 \quad 0 \quad 0}_{l=3} \quad \underbrace{1 \quad 1}_{l=2} \quad \underbrace{0}_{l=1} \quad \underbrace{1}_{l=1} \quad \underbrace{0}_{l=1} \quad \underbrace{1 \quad 1 \quad 1 \quad 1}_{l=4} \quad \underbrace{0}_{l=1}$$

2.2 The Random Walk Hypothesis I

Iterations: Stochastic model and a limit theorem.

Let $X_1, \ldots, X_n$ be iid random variables with

$$P(X_i = 1) = p, \quad P(X_i = 0) = q = 1 - p,$$

and let

$$I = \# \text{ iterations in } (X_1, \ldots, X_n).$$

Then, approximately for large n:

$$I \sim \mathsf{N}(2npq, 4npq(1 - 3pq))$$

2.2 The Random Walk Hypothesis I

Behaviour of the sign sequence.

- Now define

$$d_t = \begin{cases} 1 & \text{if } r_t > 0 \\ 0 & \text{if } r_t \leq 0 \end{cases}$$

- If the RWH I holds, (d_t) should look like generated by iid Bernoulli trials.

- Let's investigate this for
  - DJIA (Dow-Jones Industrial Average),
  - SSEC (Shanghai Composite),
  - IBM,
  - WTI crude oil.

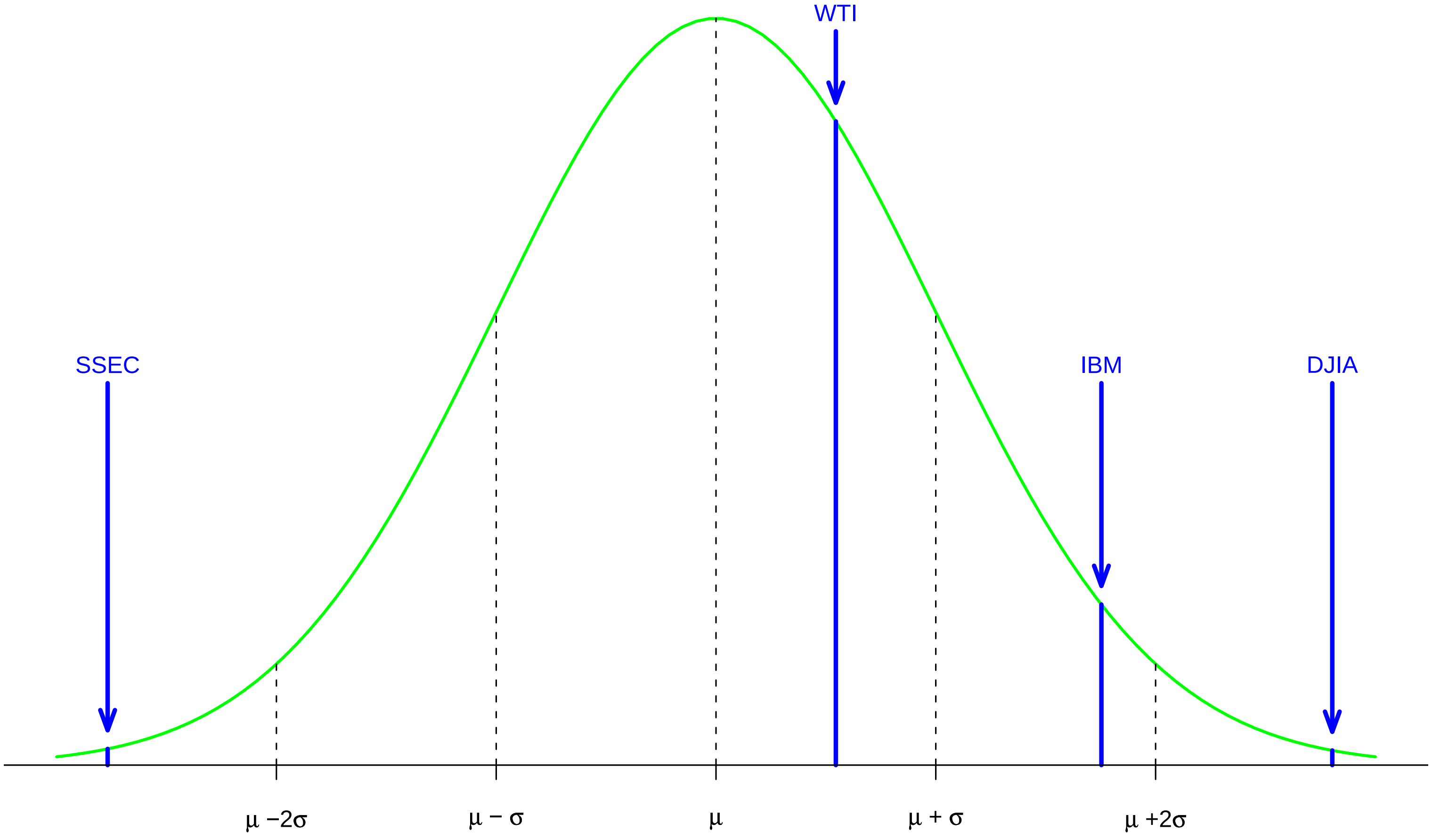
2.2 The Random Walk Hypothesis I

Example: Iterations in Asset Returns.

	DJIA	SSEC	IBM	WTI
first day last day	1998-01-02 2008-09-30	1998-01-05 2008-09-26	1998-01-02 2008-09-30	1998-01-02 2008-09-30
# observations	2703	2759	2703	2692
# iterations	1423	1306	1397	1358
expected # it. sd. # it.	1350 26.0	1379 26.3	1351 26.0	1344 26.0
p-value	0.005	0.006	0.08	0.59

2.2 The Random Walk Hypothesis I

Iterations: Their distribution under RWH I and realizations.



2.2 The Random Walk Hypothesis I

Conclusions.

- There is strong evidence against RWH I for the DJIA as well as for SSEC.
- The theory of iterations finds evidence against RWH I for neiher IBM nor WTI.
- What about a different RWH, a different statistical approach?

2.3 The Random Walk Hypothesis II

A random walk with independent asset price changes.

- The RWH II states that prices evolve according to:

$$\ln P_t = \mu + \ln P_{t-1} + \epsilon_t, \quad (\epsilon_t): \text{ independent}$$

- For returns:

$$r_t = \mu + \epsilon_t, \quad (\epsilon_t): \text{ independent}$$

- This is weaker than the RWH I.

2.3 The Random Walk Hypothesis II

Statistical test of the RWH II.

- Independence of two random variables X, Y is equivalent to

$$\mathrm{cor}(f(X), g(Y)) = 0 \qquad \text{for any choice of functions } f,\ g.$$

- Then, what about

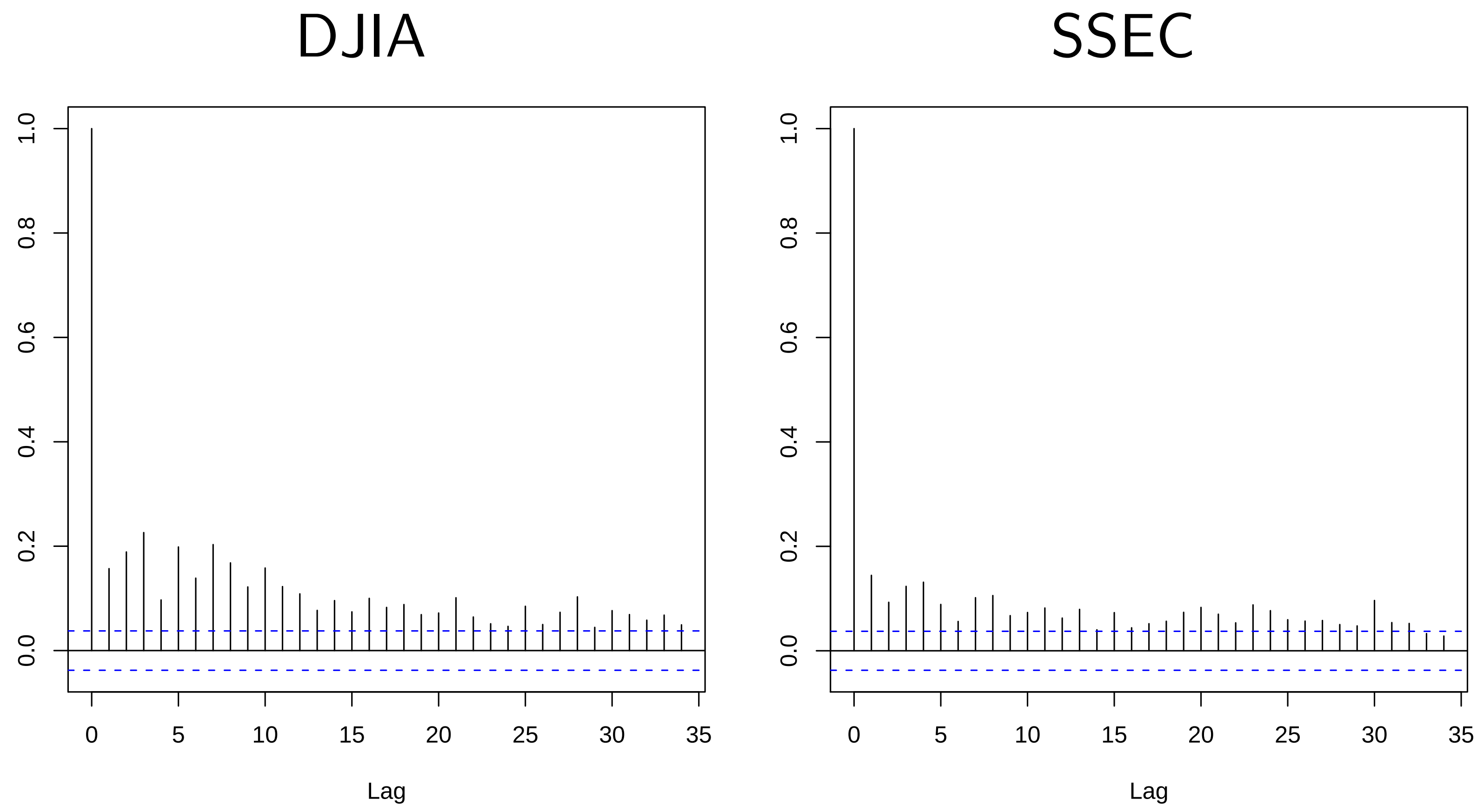
$$\mathrm{cor}(r_t^2, r_{t-1}^2),$$

the autocorrelation of squared returns?

- In case the RWH II holds, it has to be zero.
- Let's investigate this for our examples.

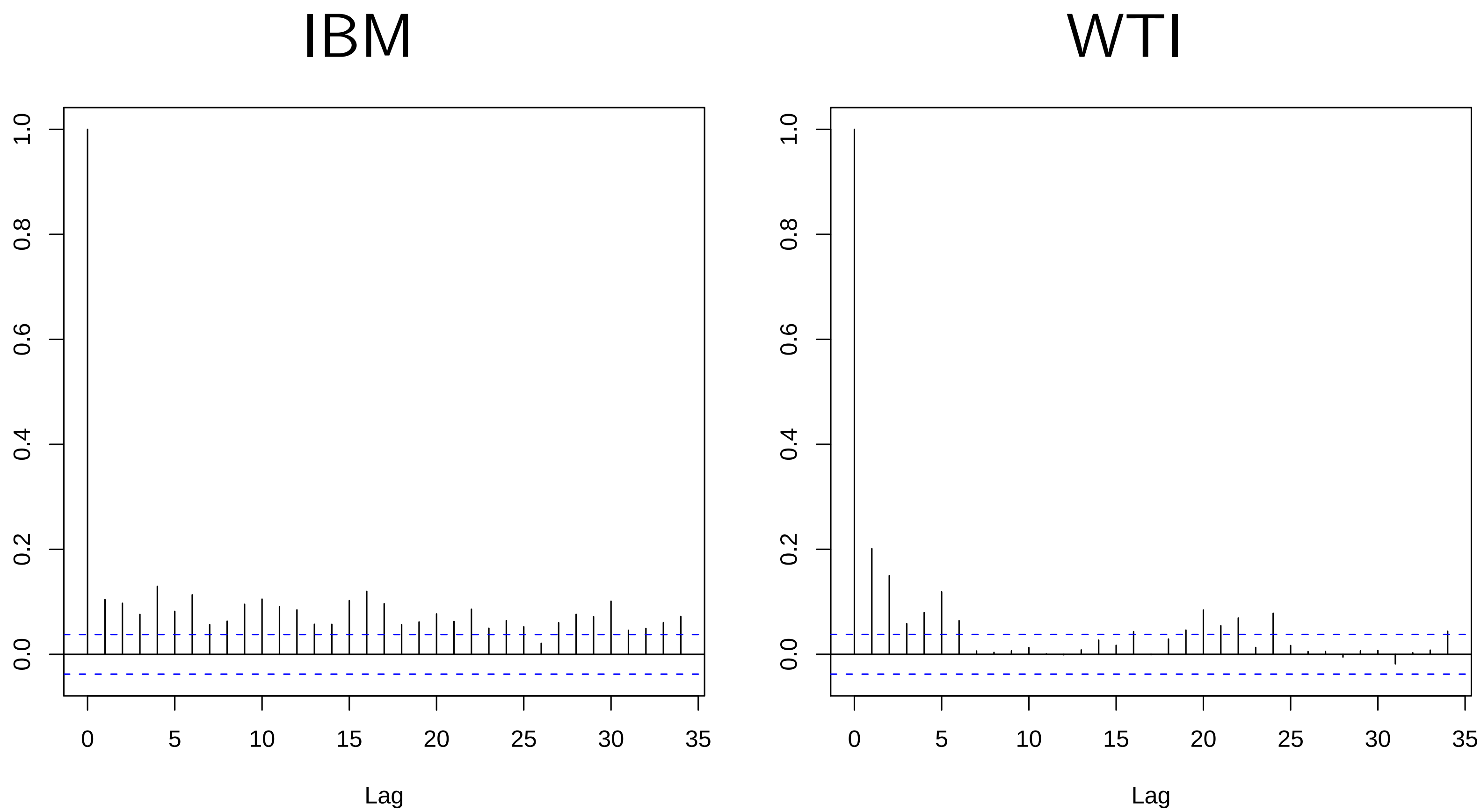
2.3 The Random Walk Hypothesis II

Autocorrelations of squared returns.



2.3 The Random Walk Hypothesis II

Autocorrelations of squared returns.



2.3 The Random Walk Hypothesis II

Conclusions.

- There is strong evidence against RWH II for all four series.
- Squares of (zero-mean) variables can be linked to the amount of variability in the variables.
- Autocorrelation in squares indicates some persistence of volatility.
- This phenomenon is the focal point of (G)ARCH processes.

2.4 The Random Walk Hypothesis III

A random walk with uncorrelated asset price changes.

- The RWH III states that prices evolve according to:

$$\ln P_t = \mu + \ln P_{t-1} + \epsilon_t, \quad (\epsilon_t): \text{ uncorrelated}$$

- For returns:

$$r_t = \mu + \epsilon_t, \quad (\epsilon_t): \text{ uncorrelated}$$

- This is weaker than the RWH II.

2.4 The Random Walk Hypothesis III

Statistical test of the RWH III.

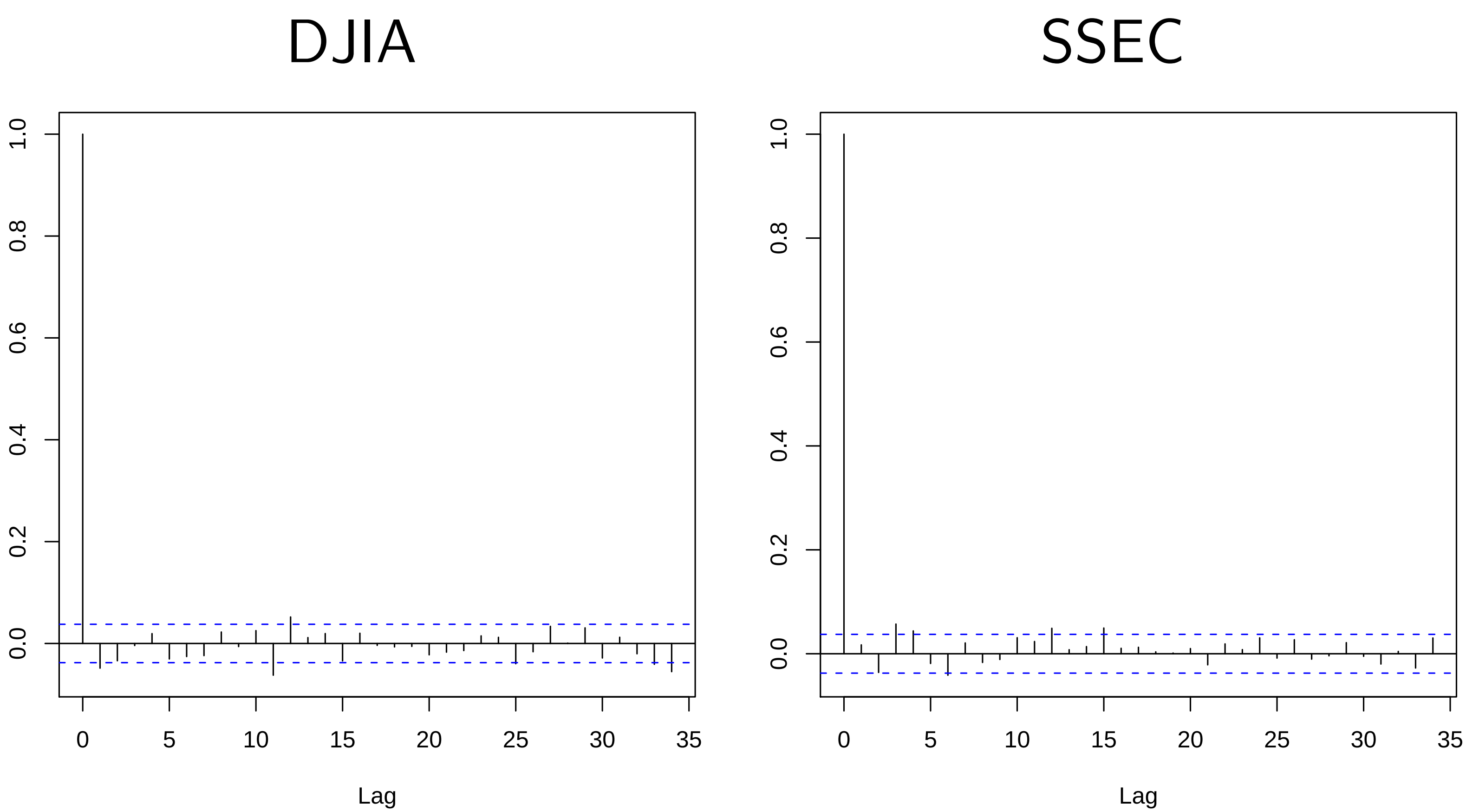
- Two random variables X, Y are uncorrelated if

$$\operatorname{cor}(f(X), g(Y)) = 0 \quad \text{for } \textit{linear} \text{ functions } f,\ g.$$

- As before: What about $\operatorname{cor}(r_t, r_{t-1})$?
- In case the RWH III holds, it has to be zero.
- Let's investigate this for our examples.

2.4 The Random Walk Hypothesis III

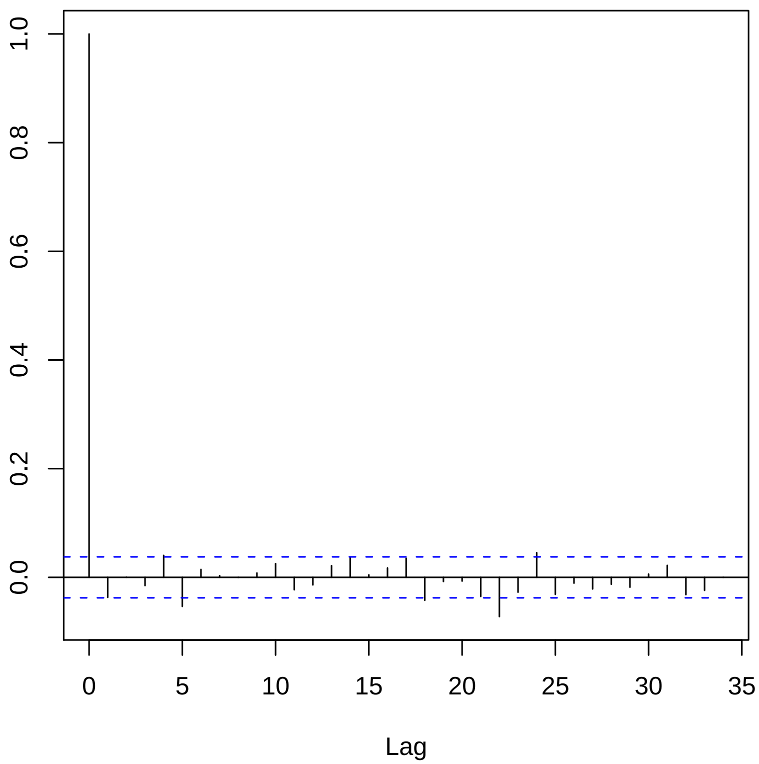
Autocorrelations of returns.



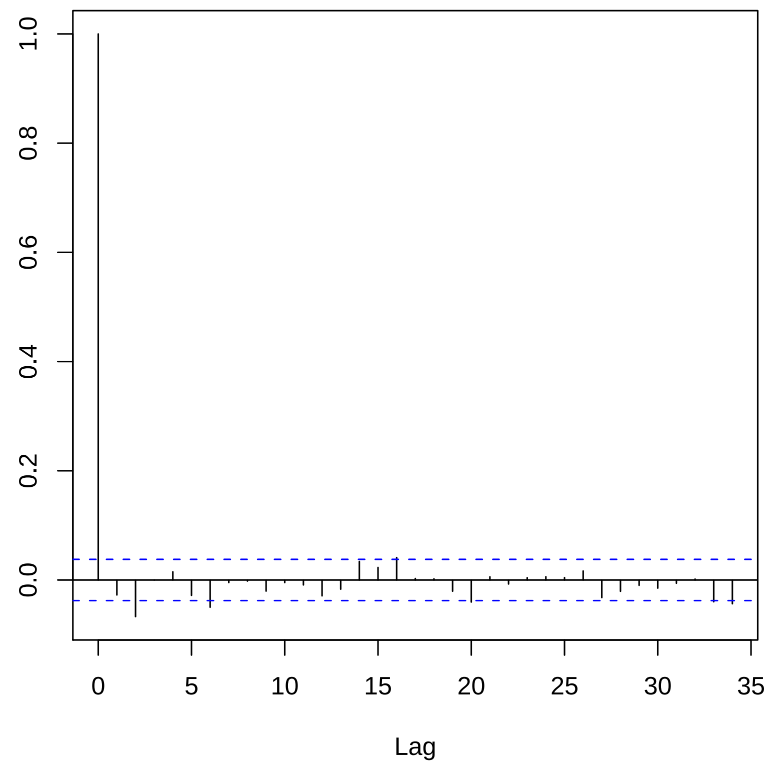
2.4 The Random Walk Hypothesis III

Autocorrelations of returns.

IBM



WTI



2.4 The Random Walk Hypothesis III

Conclusions.

- There is no consistent evidence against RWH III for any of the four series.
- This has very important consequences.
- For example, it is not possible to forecast expected returns, using past return information.
  (The conditional expectation will equal μ.)
- These ideas (and others) give rise to the use of martingale theory for asset prices.

2.5 Efficient Markets

Information.

- Market efficiency means: Information is processed efficiently.
- Which information?
- The RWH above use only internal information (from the same series).
- "Information" can be seen in a wider sense.
- Formally, we can speak of the information set. (Think of a σ-algebra.)

2.5 Efficient Markets

Different forms of market efficiency.

- Weak form efficiency.
  information set: all current and past prices.

- Semi-strong form efficiency.
  information set: all publicly available information.

- strong-form efficiency.
  information set: all information, public or private.

What about asymmetric information?

2.6 Conclusions

Predictability.

- The traces of predictability depend on the model used.
- Can you use this predictability to outsmart the market? Probably not. . . (Transaction cost etc.)
- Aspects of predictability can be very important for practical purposes. (E.g., risk evaluation.)
- Patterns in asset prices (think of data-mining) are not necessarily evidence of predictability. (They may disappear out-of-sample.)

2.6 Conclusions

The scope of approaches.

- Efficient-market theory has a strong normative component. (Theories of derivative pricing etc.)
- Efficient-market models provide very important benchmarks.
- A completely different approach: "behavioural finance".