FM 431: Econometrics of Financial Markets

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İSTANBUL BİLGİ ÜNİVERSİTESİ

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- The slides were produced using LaTeX and R (the R project; www.R-project.org) on a Linux system.
- Course material is available at www.hs-stat.com/courses/FM431.

Chapter 1:

Introduction

1.1 The Scope of This Course

Dynamic economic phenomena.

The scope of this course is to study stochastic models for dynamic economic phenomena, with a focus on financial markets.

1.1 The Scope of This Course

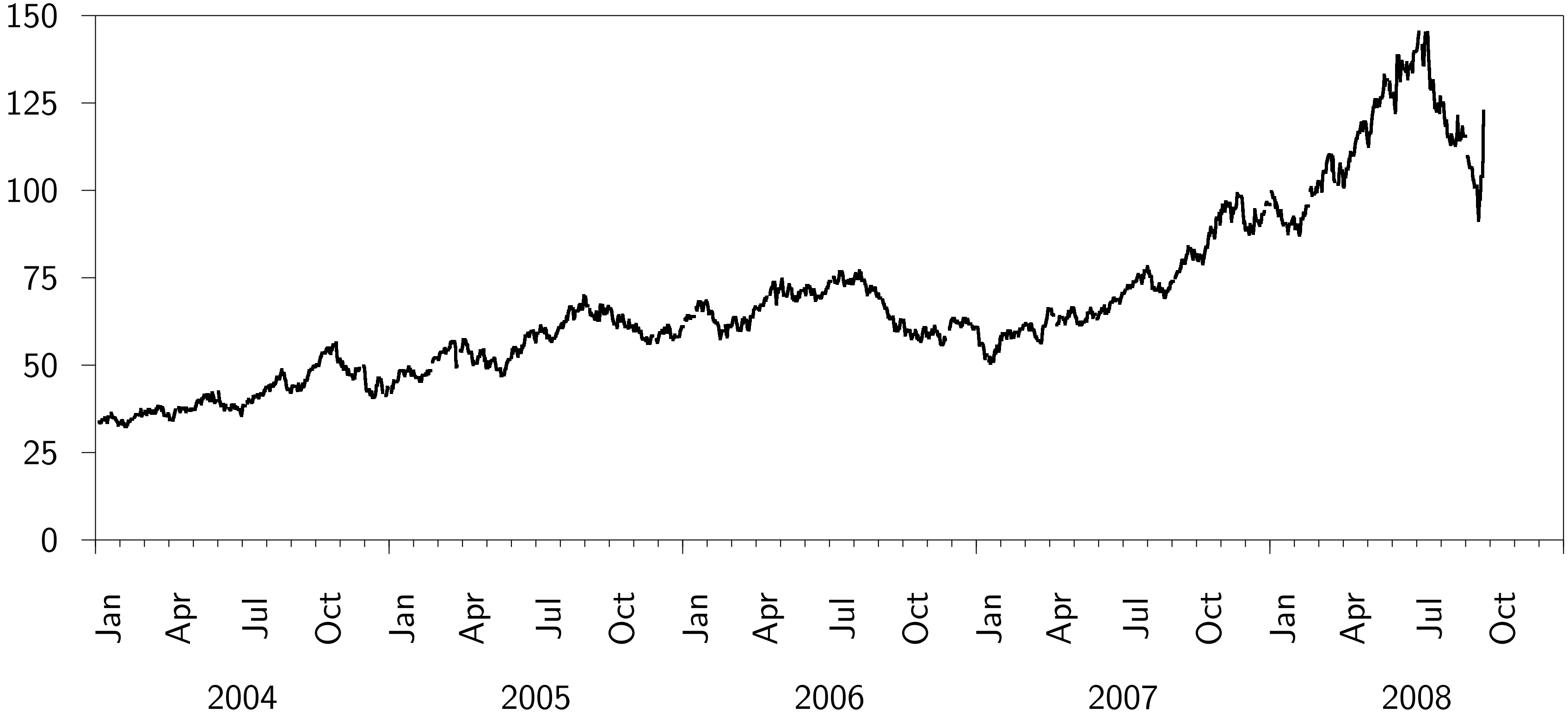
Dynamic economic phenomena.

We shall learn techniques for

- interpreting economic data,
- testing hypotheses concerning financial markets,
- forecasting.
  - What can we forecast at all?
  - Which aspects of a time series can we forecast?

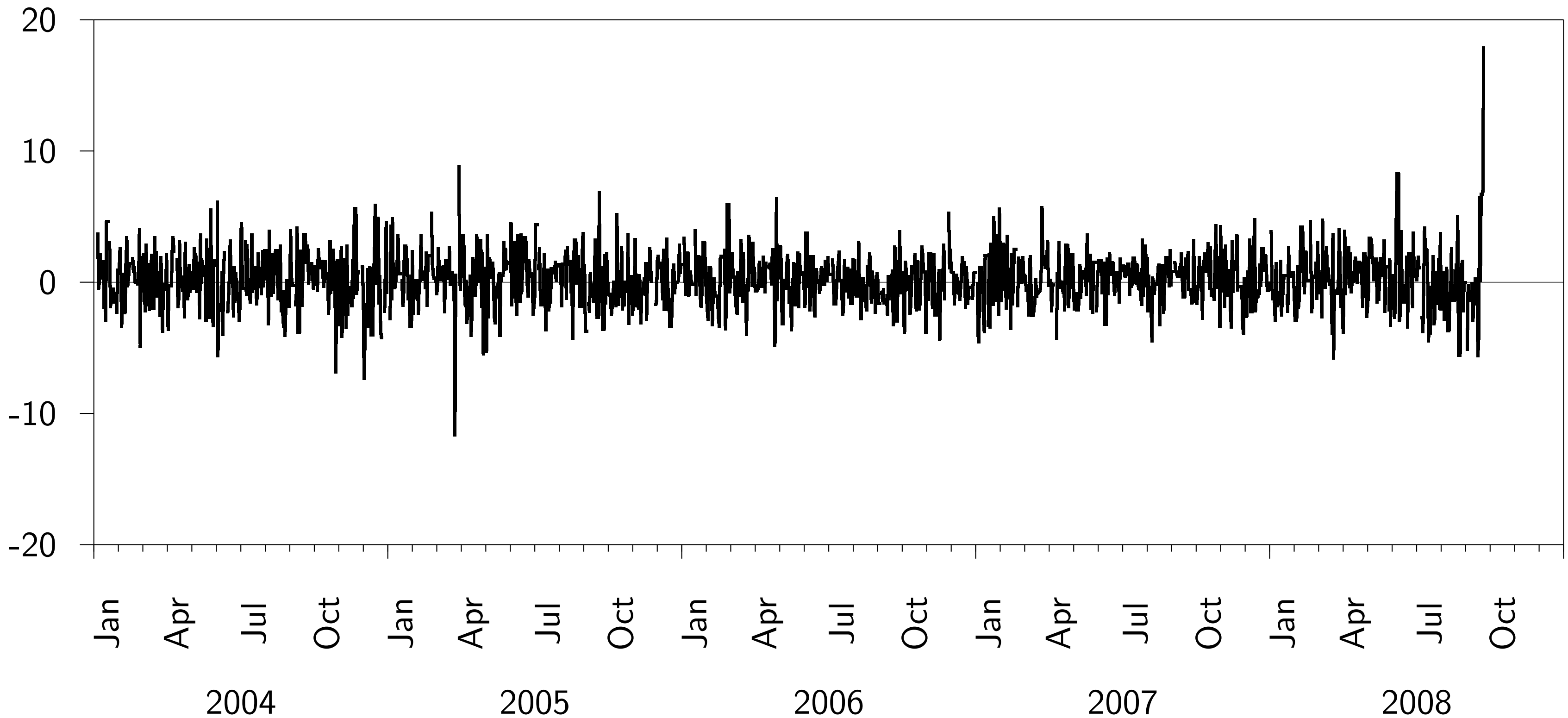
1.1 The Scope of This Course

Example: WTI crude oil, the daily level series.



1.1 The Scope of This Course

Example: WTI crude oil, the daily return series.



1.1 The Scope of This Course

Example: WTI crude oil.

We guess:

- The price process is not mean-reverting.
- The return process is heteroskedastic.

Questions:

- Which aspects of the processes can be forecast?
- In what way is the oil price process related to other economic/ financial processes?

1.2 Prerequisites

Students need to be familiar with topics from:

- descriptive statistics
- probability calculus, stochastic modeling
- inductive statistics

Furthermore, you need to know:

- correlation, partial correlation
- regression

1.2 Prerequisites

Remember this scheme from our statistics course.

OBSERVATIONS

- variables
- observed values
- relative frequency
- empirical distribution

statistical inference →

←

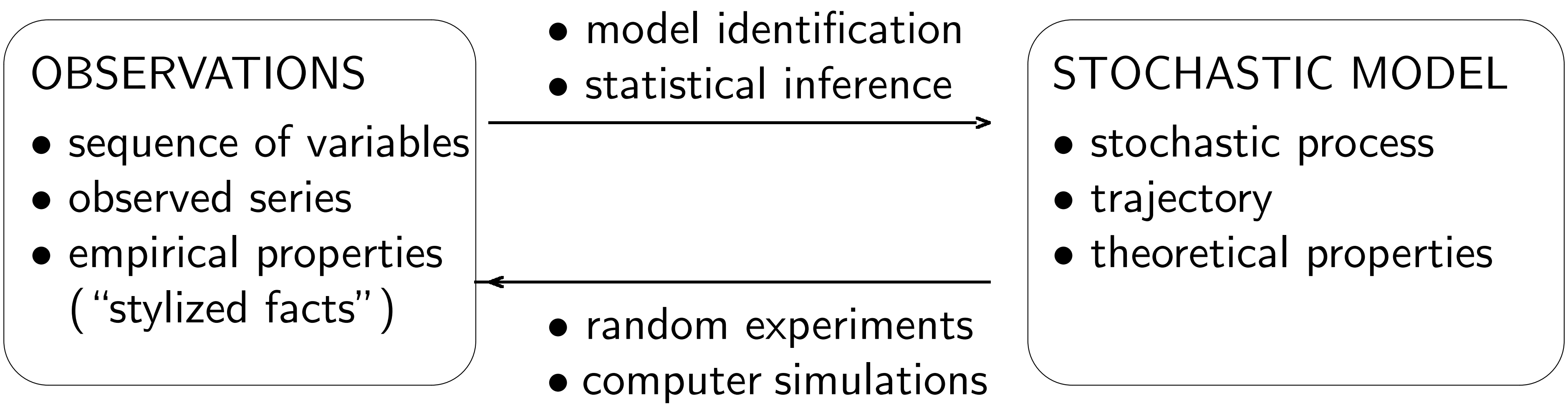
- random experiments
- computer simulations

STOCHASTIC MODEL

- random variables
- their realizations
- probability
- probability distribution

1.3 Stochastic Time Series Analysis

An adaptation of the scheme for time series analysis.



Where could we put *forecasting*?

1.3 Stochastic Time Series Analysis

Identification tools.

- Stochastic time series analysis means:
  Find a model which may have produced the data.

- Determine the empirical properties of a given series, relate them to properties of candidate models: tools are needed.

- Most important: acf, pacf

1.3 Stochastic Time Series Analysis

Some odd examples. — How were the following series created?

A) 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1,. . .

B) $+1$, $+1$, -1, $+1$, $+1$, -1, $+1$, -1, $+1$, -1, -1, -1, -1, $+1$, $+1$, $+1$, $+1$, -1, . . .

C) 0, $+1$, $+2$, $+1$, $+2$, $+3$, $+2$, $+3$, $+2$, $+3$, $+2$, $+1$, 0, -1, 0, $+1$, $+2$, $+3$, $+2$, . . .

D) 2, 6, 4, 4, 1, 4, 1, 6, 3, 3, 4, 6, 1, 6, 5, 3, 3, 1, 5, 4, 3, 2, . . .

1.3 Stochastic Time Series Analysis

Some odd examples. — How were the following series created?

E) 0.429, −1.176, −0.643, 0.799, −0.587, −1.723, 1.989, −0.189, 2.903, 3.860, −2.384, −3.638, −0.289, 0.276, 1.594, −2.708, −3.213, −1.916, 1.306, −0.828, −2.596, 2.071, 0.079, 1.327, −0.552, −0.617, −0.639, −0.751, 2.982, 2.590, 1.098, 2.045, −0.238, 0.293, 2.545, −1.767, 0.221, 0.446, 1.581, 0.748, 0.532, 0.965, −1.124, 2.624, 2.014, −0.401, −0.377, 2.386, 1.583, 0.547, 0.723, 1.444, −0.544, −1.224, 0.380, −2.829, 1.883, −1.205, 0.341, −0.631, 0.226, 0.645, −0.608, 1.274, 0.374, 1.477, 0.564, −0.315, 0.524, 1.573, −0.150, 0.033, 2.198, 1.714, 0.870, 0.439, 4.113, 1.027, −3.299, 2.399, 1.880, −1.381, 1.330, 2.952, 0.083, −1.024, −0.935, −0.671, 1.481, −0.905, −1.367, 1.419, −0.452, −1.815, 0.627, 1.486, −1.623, −1.223, 0.681, 3.555, 1.947, 0.962, 0.047, −0.954, 0.623, 0.611, 1.901, −1.493, 1.015, −1.268, −1.119, −0.796, −4.581, −3.530, 1.235, −0.797, 4.290, −3.245, 0.769, 1.884, 1.446, −0.586, −0.060, −0.159, 0.457, −2.823, −1.302, −2.267, 1.833, 1.023, −0.071, 2.603, 0.097, 0.040, −0.857, 1.312, −2.676, −0.485, −0.898, −1.378, 0.698, −0.832, 2.399, 2.024, 0.321, 2.742, 0.449, 0.164, −1.338, −2.220, 0.293, 0.342, 0.870, 0.527, −0.888, −0.087, 1.131, −0.335, −2.164, 0.262, −3.986, −4.055, 1.926, −1.474, −1.967, −8.306, 2.433, −3.048, 1.744, 5.073, 0.817, −2.693, 0.717, 0.690, 2.319, −3.364, −1.316, −2.015, −3.734, −1.517, −1.357, −5.668, 1.591, 2.212, 1.251, 2.231, −0.593, −2.426, 1.383, −3.204, −3.567, 1.627, 2.269, 2.481, 4.179, 0.009, 2.890, −3.005, 0.887, 1.410

1.4 Stochastic Processes

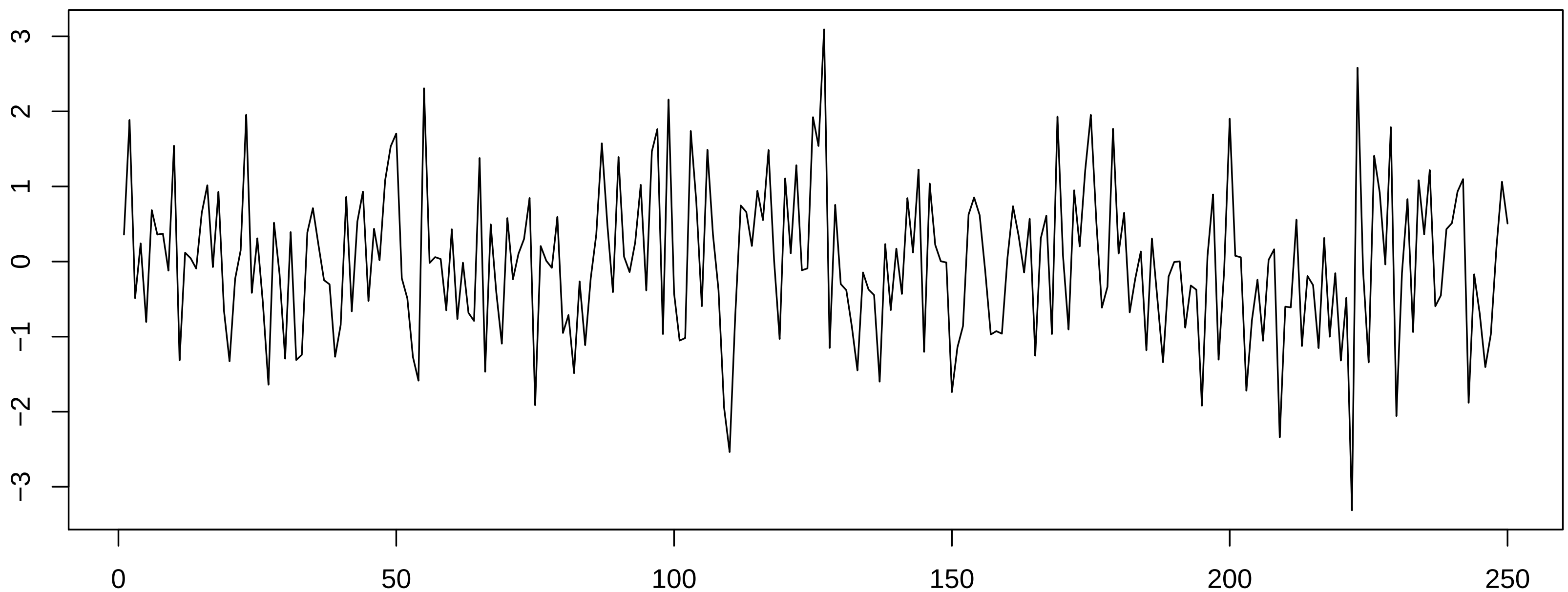
Definition:

- A sequence $(X_t)_{t \in B}$ of random variables X_t with index set $B \subset \mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$ is called a discrete-time stochastic process.
- A realization $t \mapsto X_t(\omega)$ is called a trajectory or a sample path.

1.4 Stochastic Processes

Example: Gaussian white noise.

This is defined as $(X_t)_{t=0,1,\ldots}$ with $X_t \sim \mathrm{N}(0, \sigma^2)$ iid.



1.4 Stochastic Processes

Example: Random walk.

This is defined as $(X_t)_{t=0,1,\ldots}$ with $X_t = X_{t-1} + \epsilon_t$, $X_0 = 0$, (ϵ_t): white noise.

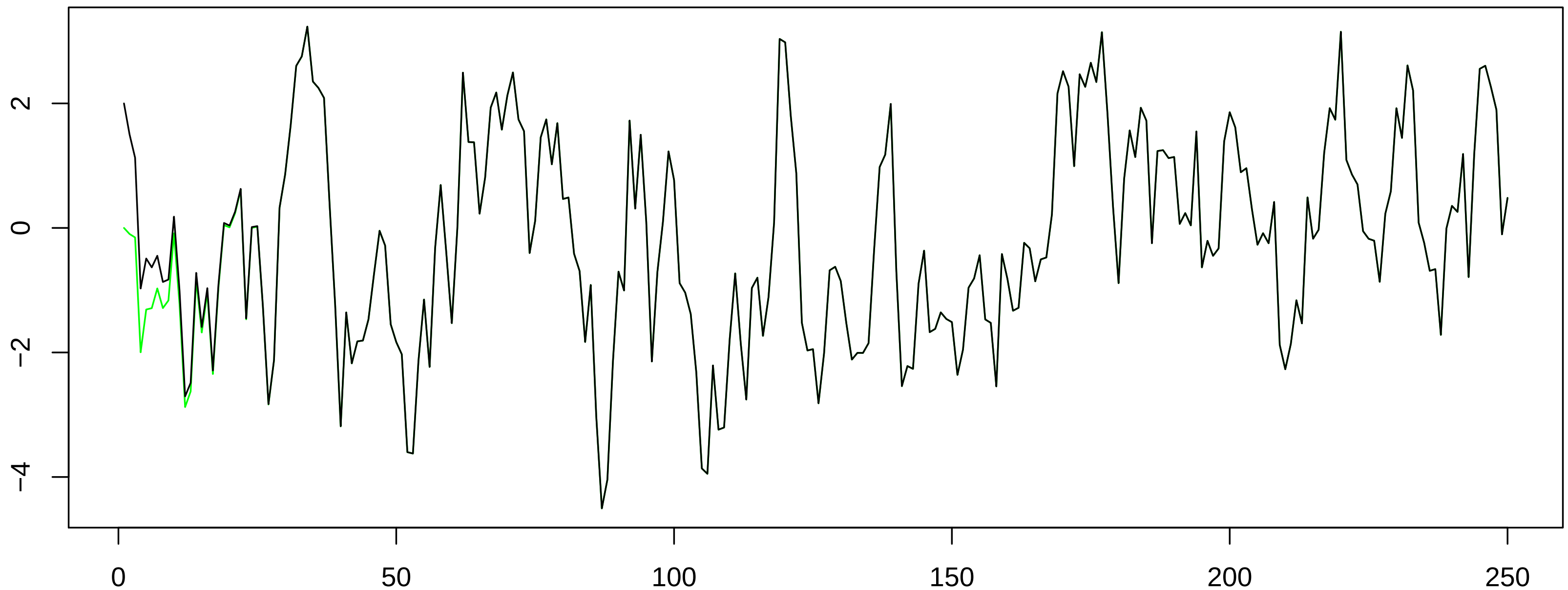
A simulation with Gaussian white noise:

1.4 Stochastic Processes

Example: The AR(1) process.

This is defined as $(X_t)_{t=0,1,\ldots}$ with $X_t = c + \alpha X_{t-1} + \epsilon_t$,
(ϵ_t): white noise.

A simulation with $c = 0, \alpha = 0.8$, and Gaussian white noise:

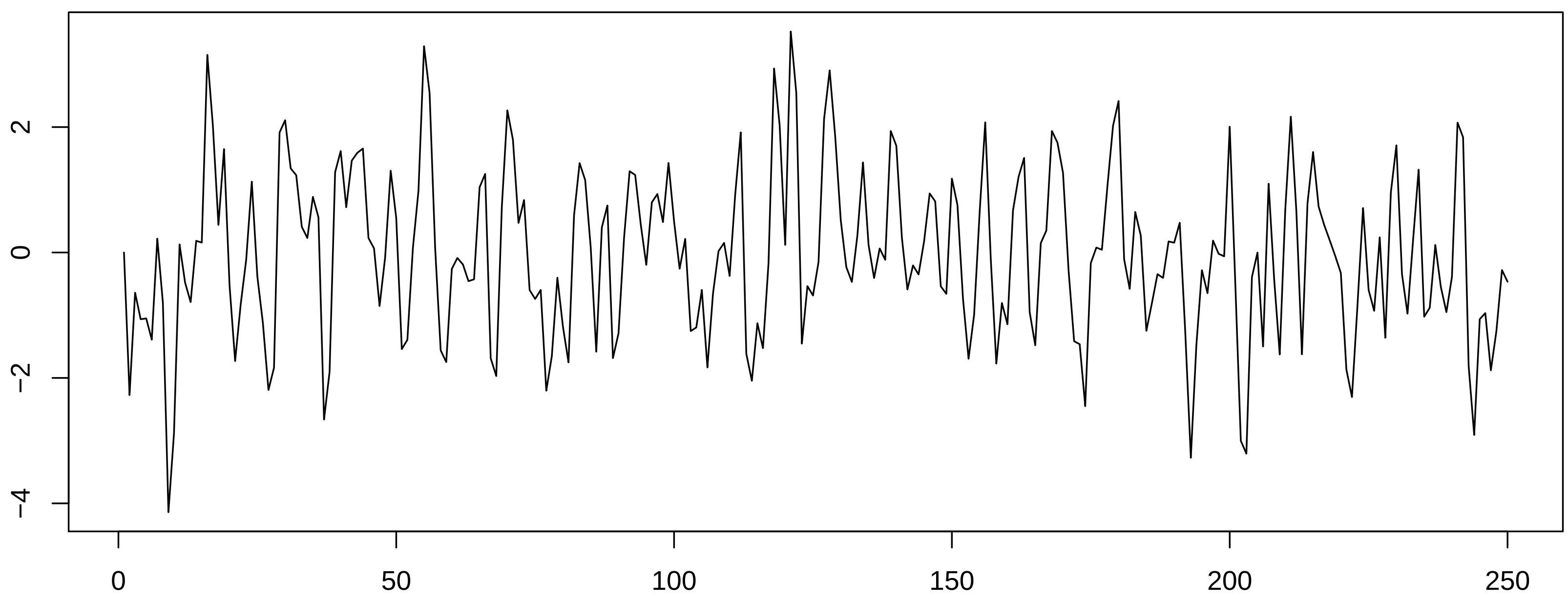


1.4 Stochastic Processes

Example: The MA(1) process.

This is defined as $(X_t)_{t=0,1,\ldots}$ with $X_t = c + \epsilon_t + \beta\epsilon_{t-1}$, (ϵ_t): white noise.

A simulation with $c = 0, \beta = 0.8$, and Gaussian white noise:



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1.4 Stochastic Processes

Stationarity. A process is called stationary if its statistical properties are time-invariant. — **Definition:**

- (X_t) is called weakly stationary if, for all s, t,

$$\mathsf{E}(X_t) \equiv \mu, \quad \mathsf{cov}(X_t, X_{t+s}) \equiv \gamma(s).$$

- Autocovariance function of (X_t):

$$s \mapsto \gamma(s) = \mathsf{cov}(X_t, X_{t+s})$$

- Autocorrelation function of (X_t):

$$s \mapsto \rho(s) = \mathsf{cor}(X_t, X_{t+s}) = \gamma(s)/\gamma(0)$$

1.4 Stochastic Processes

Autocorrelation at lag s.

- For a stochastic process (X_t):

$$\rho(s) = \frac{\text{cov}(X_t, X_{t+s})}{\text{var}(X_t)} = \frac{\gamma(s)}{\gamma(0)}$$

- For a given observed series $(x_1, \ldots, x_n)$:

$$r(s) = \frac{\frac{1}{n-s}\sum_{t=1}^{n-s}(x_t - \bar{x})(x_{t+s} - \bar{x})}{\frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x})^2}$$

1.4 Stochastic Processes

The correlogram.

- For a stochastic process (X_t), a plot of the function

$$s \mapsto \rho(s), \quad s = 0, 1, 2, \ldots$$

is called the correlogram of (X_t).

- For a given observed series $(x_1, \ldots, x_n)$, a plot of the function

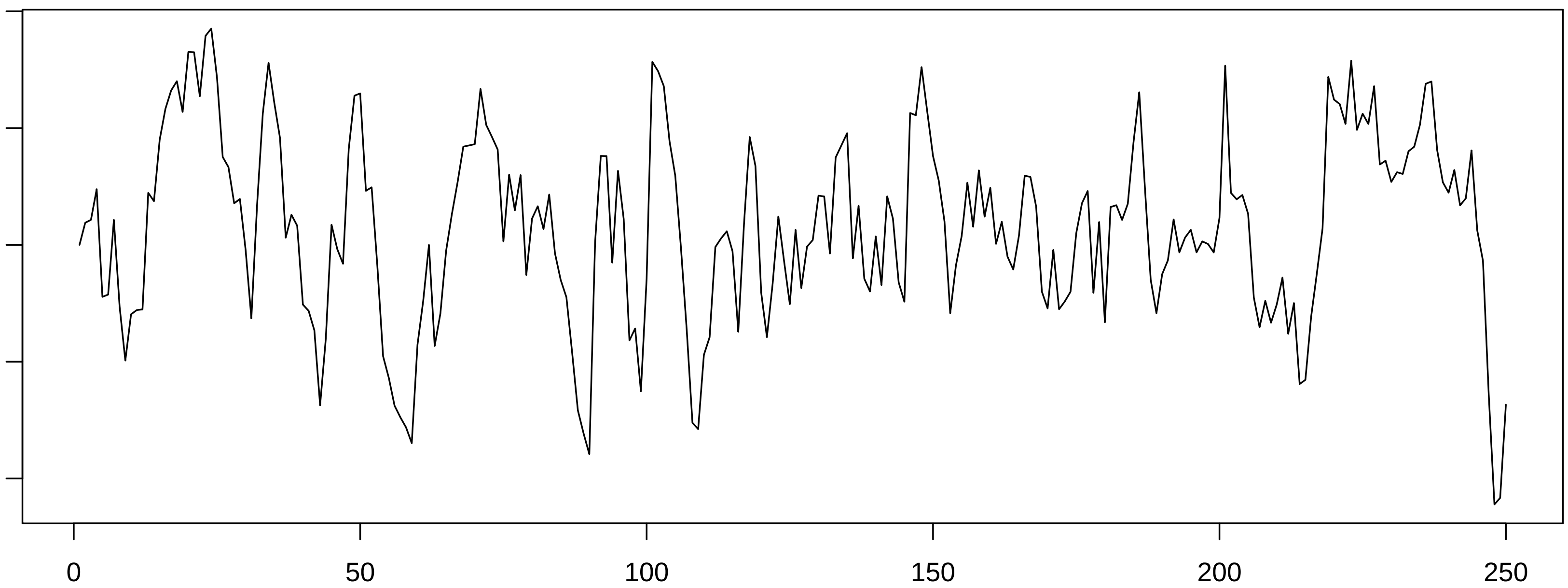
$$s \mapsto r(s), \quad s = 0, 1, 2, \ldots$$

is called the empirical correlogram of $(x_1, \ldots, x_n)$.

1.4 Stochastic Processes

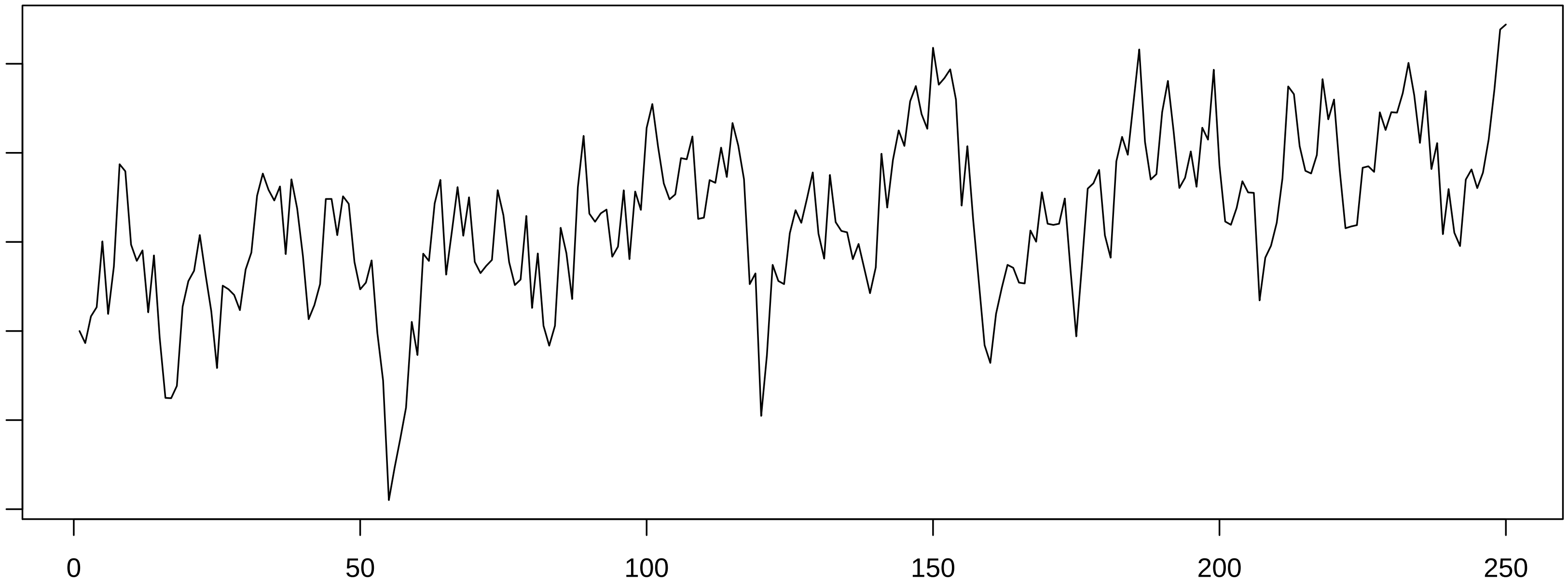
Could this process be stationary?

(What does this question mean???)



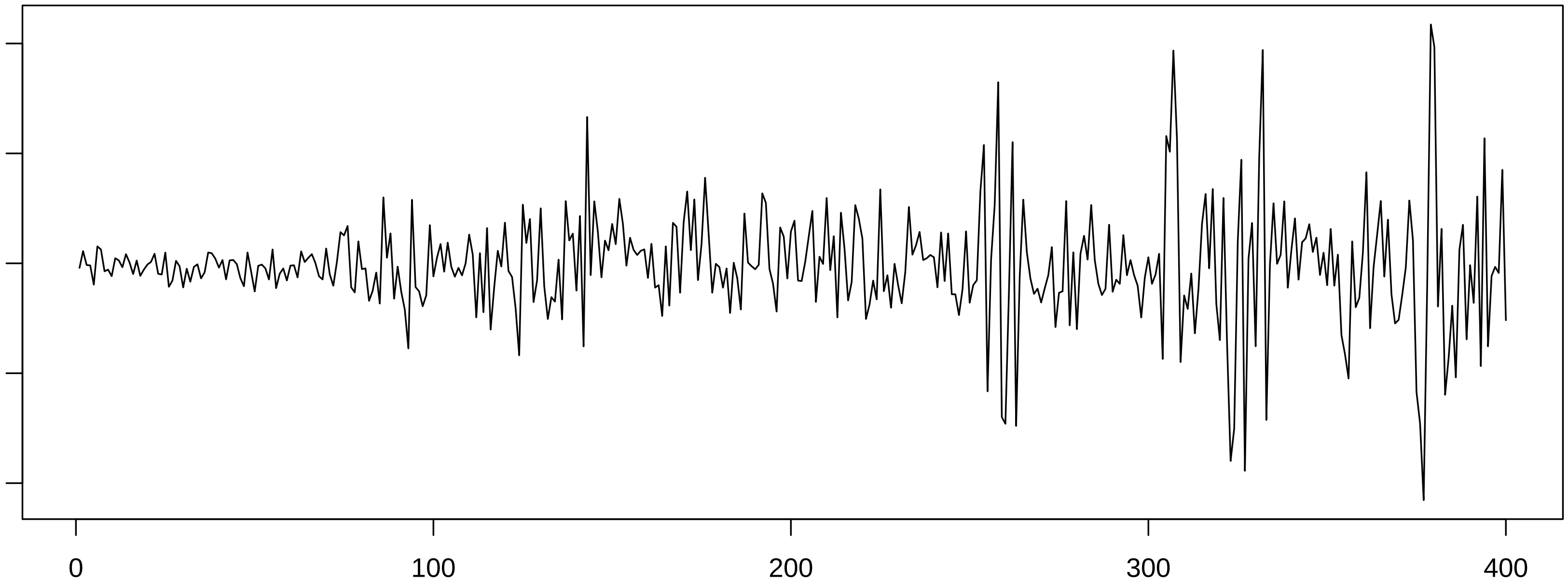
1.4 Stochastic Processes

Could this process be stationary?



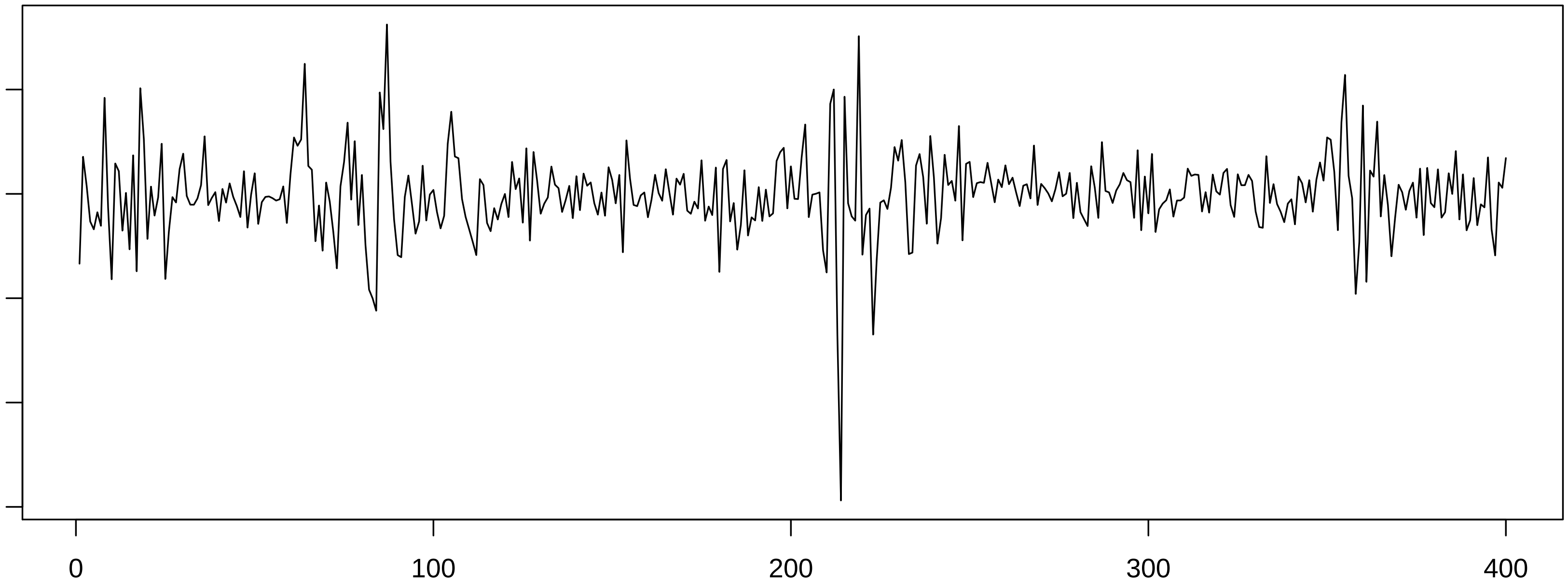
1.4 Stochastic Processes

Could this process be stationary?



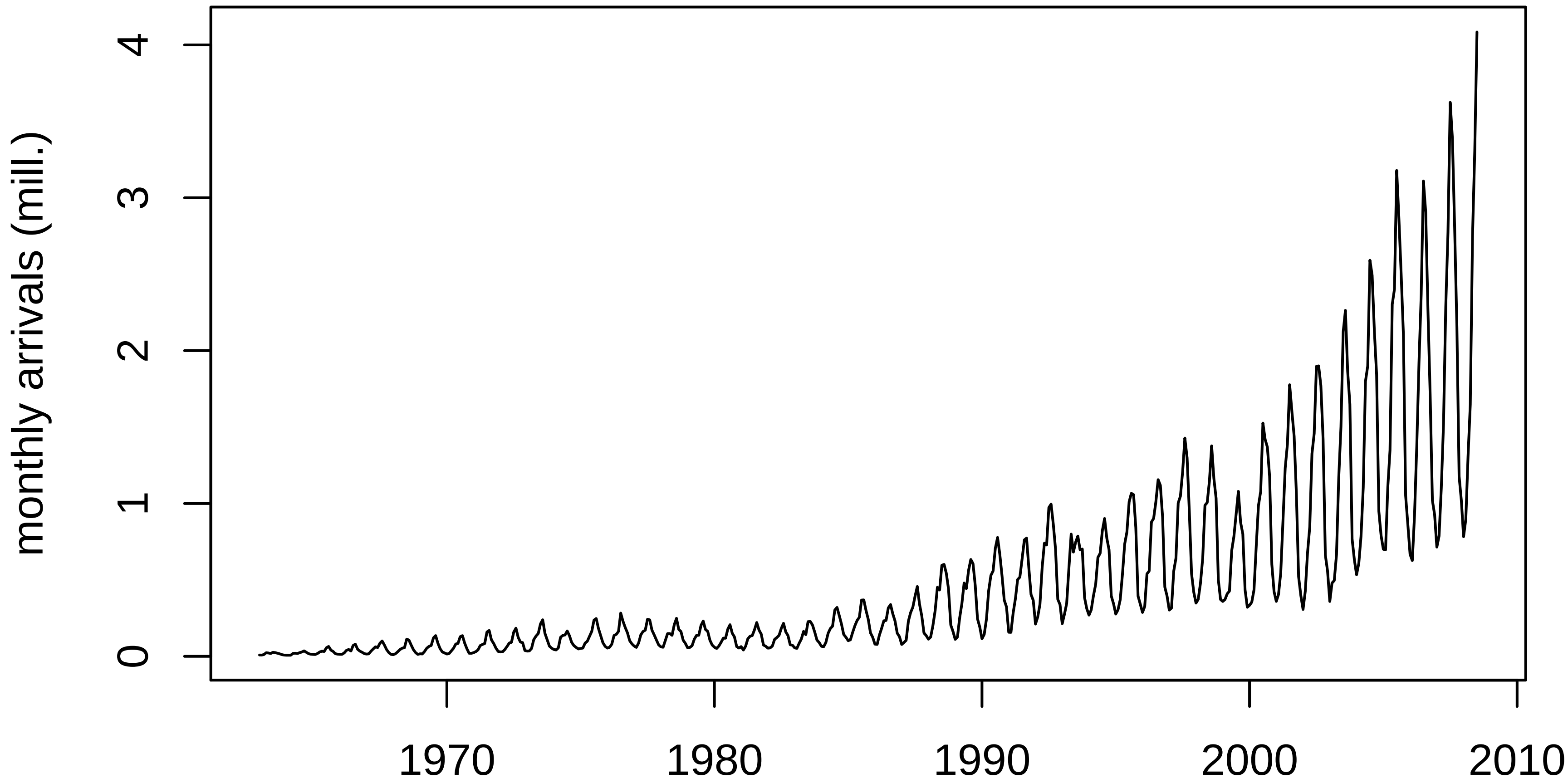
1.4 Stochastic Processes

Could this process be stationary?



1.5 Decomposing a Time Series

Monthly tourist arrivals to Turkey.



1.5 Decomposing a Time Series

A non-stochastic method.

- A non-stochastic method may be useful to analyze a series like this.
- Advantages / disadvantages?
- Idea:

$$\text{observed}_t = \text{trend}_t + \text{seasonal}_t + \text{remainder}_t$$

or

$$\text{observed}_t = \text{trend}_t \times \text{seasonal}_t \times \text{remainder}_t$$

1.5 Decomposing a Time Series

Decomposition methodology.

- Smooth the given series.
  (Linear filter, moving averages, local regression)

- Find the Jan (Feb etc.) seasonal component by averaging

  $$\text{seasonal}_t + \text{remainder}_t = \text{observed}_t \text{ - } \text{trend}_t$$

  for all t=Jan (Feb etc.)

- Compute the remainder for each month as

  $$\text{remainder}_t = \text{observed}_t - \text{trend}_t - \text{seasonal}_t$$

(For some more details, see the handout.)

1.5 Decomposing a Time Series

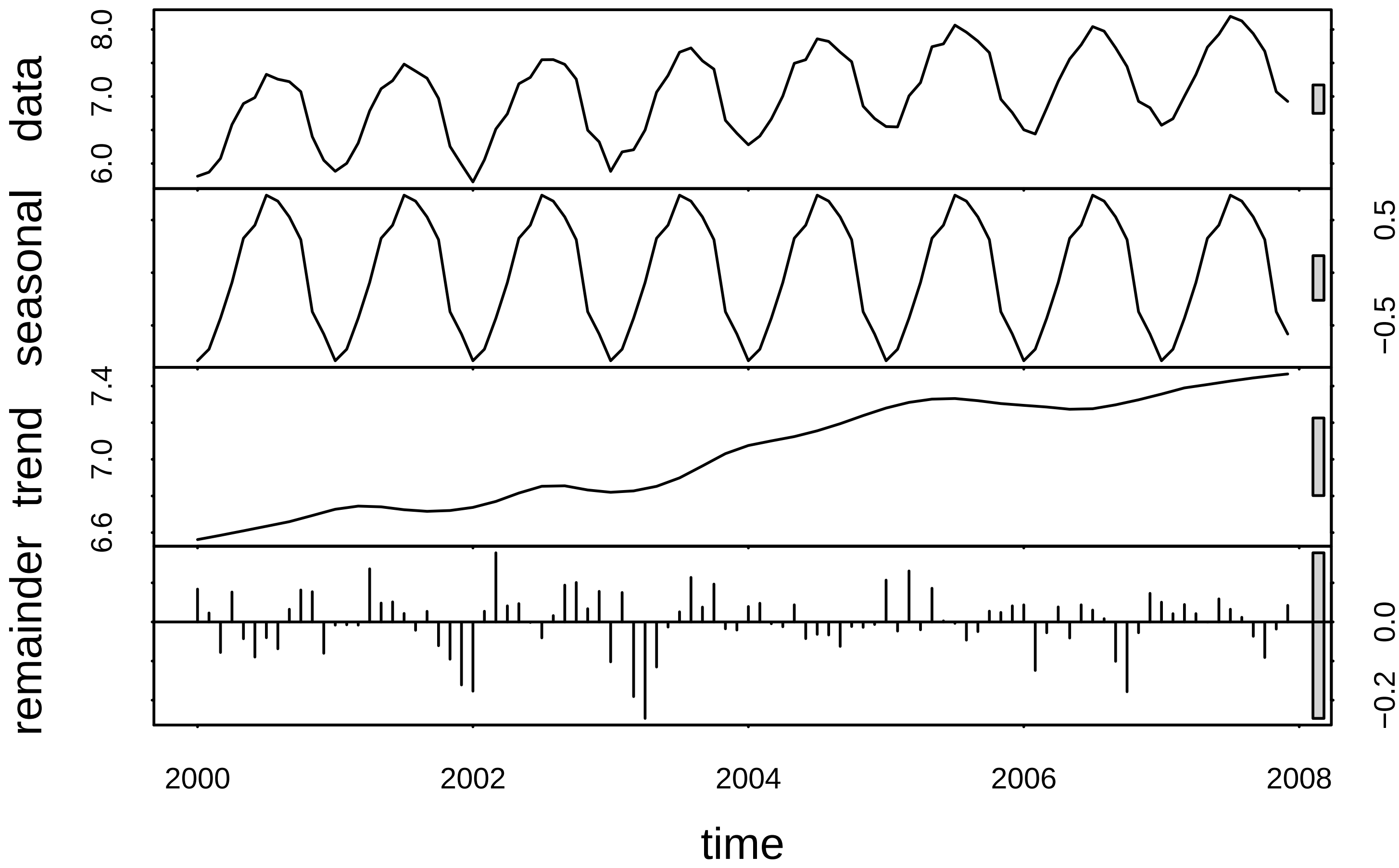
Decomposing the tourist arrival series.

We shall now:

- Decompose the logged series, using values 2000 through 2007, using stl,
- fit a regression line to the trend,
- extrapolate the trend; add on the seasonal component,
- thus obtain a forecast for the logged series (and the series itself).

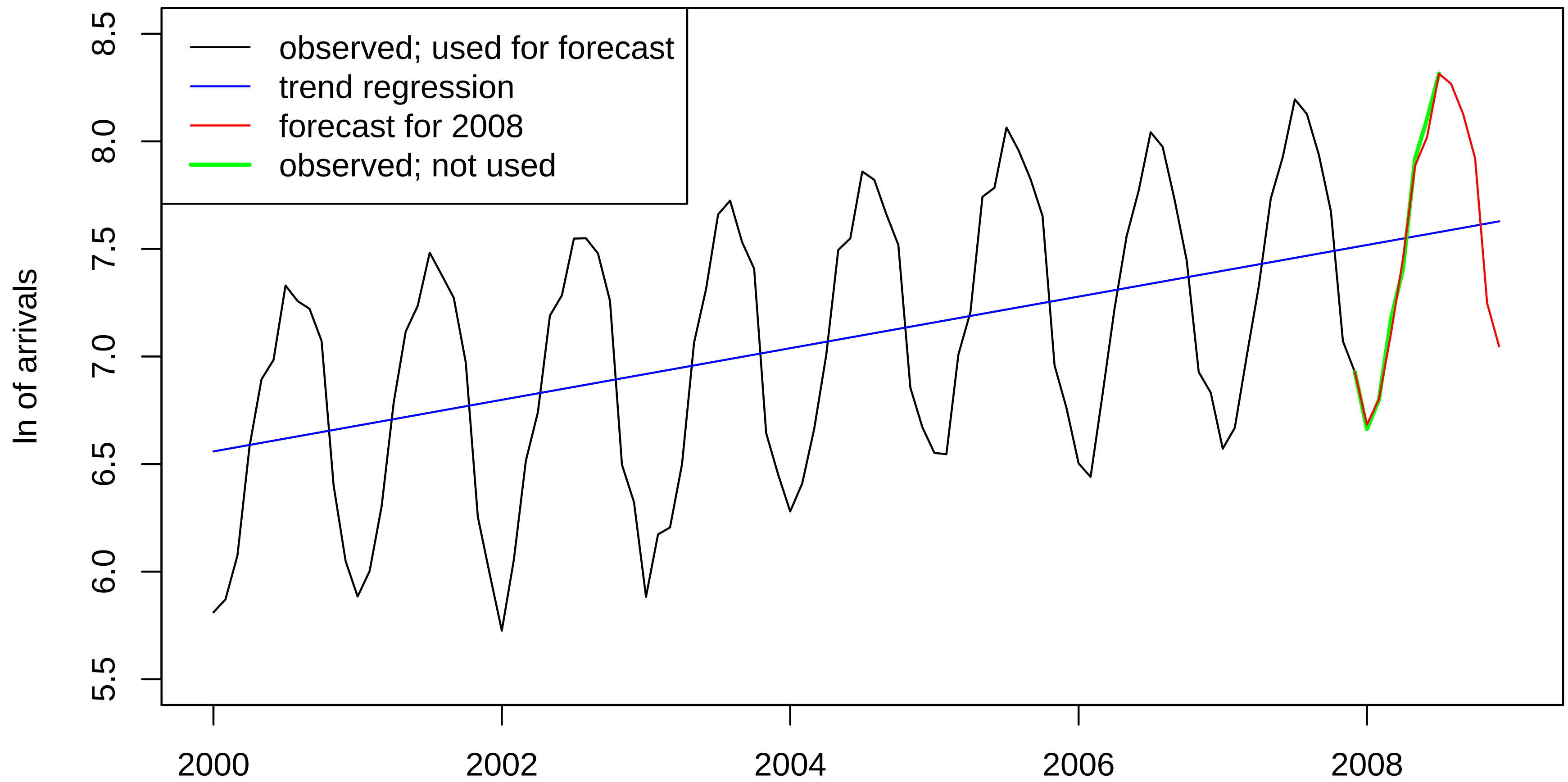
1.5 Decomposing a Time Series

Decomposition by stl.



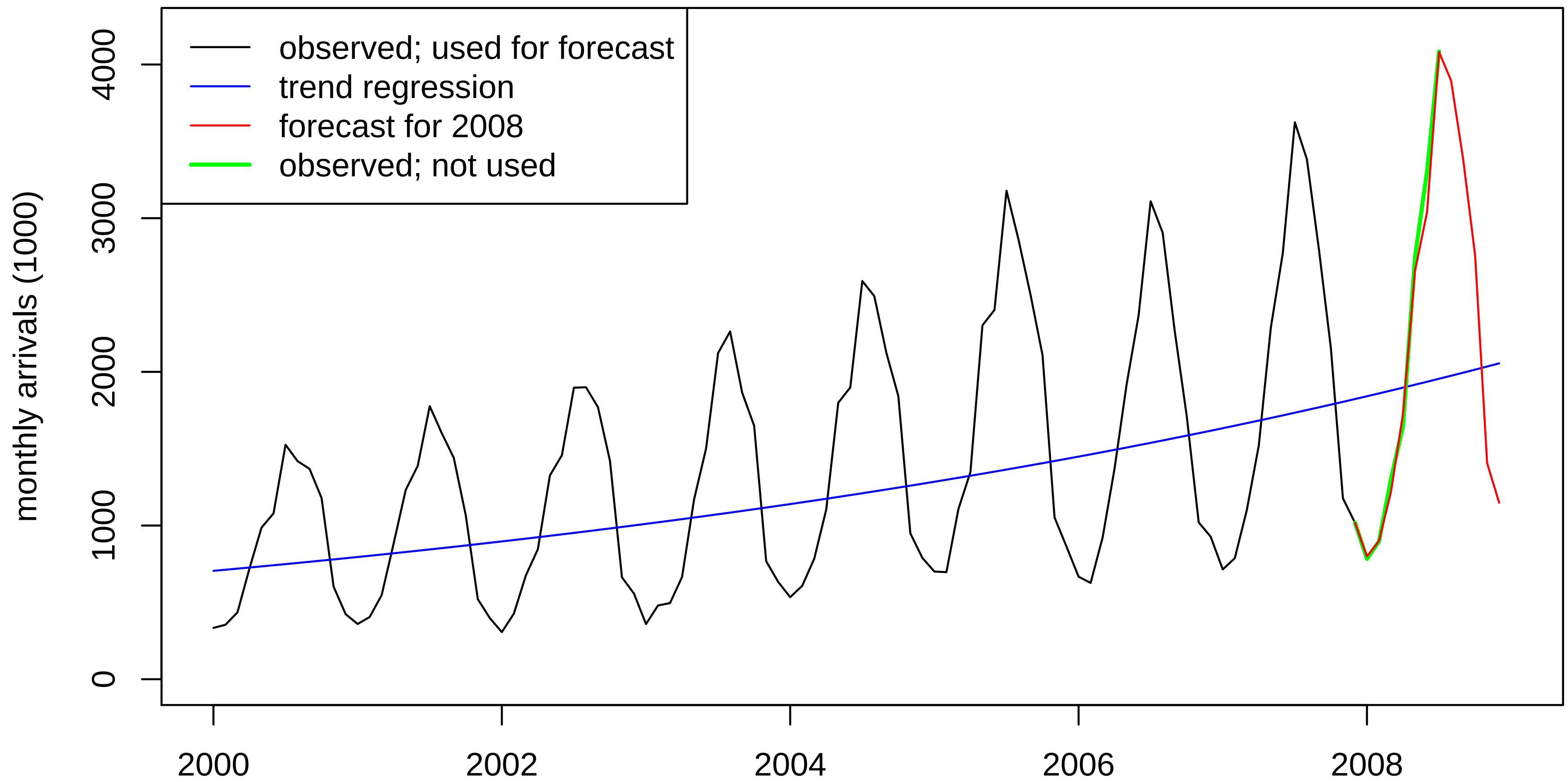
1.5 Decomposing a Time Series

The result for the logged series.



1.5 Decomposing a Time Series

The result for the series itself.



1.5 Decomposing a Time Series

Decomposing the tourist arrival series.

Questions:

- How is it possible that the fit between
  - the observed values for Jan–July 2008 (values published be the Turkish Ministry of Culture and Tourism) and
  - our forecast

  is so good???

- Who counts incoming tourists?

- How are they counted?

1.6 Outlook

Our goals and how to achieve them.

- The aim of this course is:
  - to give a systematic introduction to certain stochastic models.
  - to show applications of the models to real-world data.
- This requires a PC and software.
- **We do not tolerate the use of unlicensed software.**
- We shall show how to use R (www.R-project.org) for our purposes.

1.6 Outlook

Chapters:

1. Introduction
2. Are Asset Returns Predictable?
3. ARMA Processes
4. GARCH Processes
5. Models With Trend
6. Multi-Equation Time Series Models
7. Cointegration and Error Correction Models
8. Multivariate GARCH Processes