FM 431: Econometrics of Financial Markets

Fall 2009

PROBLEM SHEET # 4

Problem 1: Consider the model $X_t = aX_{t-1} + \epsilon_t + 0.7\epsilon_{t-1}$, where (ϵ_t) is Gaussian white noise with variance 3.

a) Which model is this?

b) Show that the process (X_t) has a unit root when $a = 1$.

c) Now suppose this process (with $a = 1$) is started in $X_1 = 0$. Compute $\text{var}(X_2)$, $\text{var}(X_3)$, $\text{var}(X_4)$.

d) What happens with $\text{var}(X_t)$ as $t \to \infty$? Give reasons for your answer.

e) Will the process in (c) become stationary as $t \to \infty$? Give reasons for your answer.

Problem 2: The following plot shows a plot of the autocorrelation function (acf) and a plot of the partial autocorrelation function (pacf) of an ARMA process.

a) Which ARMA process might that be? Give reasons for your answer.

b) Explain briefly why the acf and pacf are important when working with ARMA models in practice.

Problem 3: A time series of monthly sales was found to follow an AR(1) process (X_t) with $X_t = 10000 + 0.65X_{t-1} + \epsilon_t$, where (ϵ_t) is white noise with $\epsilon_t \sim \text{N}(0, 7000^2)$.

a) November sales were 20000 units. Compute a forecast for December sales.

b) Using this model, find an *interval* such that December sales will be within this interval with a probability of about 2/3.

c) Given the *additional* information that October sales were 27000, how would your forecast in (a) change? Give reasons for your answer.

d) Assuming that no change in the model occurs, how would you forecast sales for December *next year*?