FM 431: Econometrics of Financial Markets

Fall 2009

PROBLEM SHEET # 1

Problem 1: File `gold_silver_platinum_errors.txt` contains three time series: daily closing fixings of gold, silver, and platinum, in USD per troy ounce. Unfortunately, five typical errors have crept into this file.

a) Find the errors and fix them in a plausible way.

b) Compute daily returns in percent for each series.

c) Make a scatterplot of daily returns of gold and silver, and compute the correlation.

d) Make a scatterplot of r_t versus r_{t-1}, where r_t is the daily return of gold. Compute the correlation. What does this correlation indicate?

Problem 2: Consider the AR(1) process $y = (y_t)$, defined as

$$y_t = a_0 + a_1 y_{t-1} + \epsilon_t, \tag{1}$$

where $|a_1| < 1$ and (ϵ_t) is a sequence of iid random variables.

a) Given an initial condition y_0 for y, show that

$$y_t = a_0 \frac{1 - a_1^t}{1 - a_1} \; + \; a_1^t y_0 \; + \; \sum_{i=0}^{t-1} a_1^i \epsilon_{t-i} \tag{2}$$

solves (1).

b) Show that (2) can be obtained from (1) be recursive substitution.

c) What happens to (2) as $t \to \infty$?

d) In the absence of an initial condition, show that

$$y_t = \frac{a_0}{1 - a_1} \; + \; \sum_{i=0}^{\infty} a_1^i \epsilon_{t-i} \tag{3}$$

solves (1).

Definition: The sequence $(x_{j+1}, \ldots, x_{j+l-1}, x_{j+l})$ is called an *iteration* (or a *run*) if

$$x_j \neq x_{j+1} = \ldots = x_{j+l-1} = x_{j+l} \neq x_{j+l+1}.$$

The number l is called the *length of the iteration*.

Example: The sequence $a, a, b, b, b, a, a, b, a, b, a, a, a, a, b$ has eight iterations:

$$\underbrace{a\ a}_{l=2}\ \underbrace{b\ b\ b}_{l=3}\ \underbrace{a\ a}_{l=2}\ \underbrace{b}_{l=1}\ \underbrace{a}_{l=1}\ \underbrace{b}_{l=1}\ \underbrace{a\ a\ a\ a}_{l=4}\ \underbrace{b}_{l=1}$$

Problem 3: File `data_dji_daily.dat` contains daily closing quotes p_t and daily returns r_t of the Dow-Jones Industrial Average. Construct a new daily sequence (d_t):

$$d_t = \begin{cases} -1 & \text{if } r_t < 0, \\ +1 & \text{if } r_t \geq 0. \end{cases}$$

Count the number of iterations in (d_t). Also determine the distribution of the length of the iterations.