Harald Schmidbauer İstanbul Bilgi Üniversitesi

FM 431: Econometrics of Financial Markets

Fall 2008

Smoothing and Decomposing a Seasonal Time Series

Example: We compute moving averages for the following series:

12.3	12.9	13.6	14.4	15.3	16.3	17.2	18.0

$p = 4$ (i.e. $k = 2$):

$$x_3^* = \tfrac{1}{4}(\tfrac{1}{2} \cdot 12.3 + 12.9 + 13.6 + 14.4 + \tfrac{1}{2} \cdot 15.3),$$
$$x_4^* = \tfrac{1}{4}(\tfrac{1}{2} \cdot 12.9 + 13.6 + 14.4 + 15.3 + \tfrac{1}{2} \cdot 16.3),$$
$$x_5^* = \tfrac{1}{4}(\tfrac{1}{2} \cdot 13.6 + 14.4 + 15.3 + 16.3 + \tfrac{1}{2} \cdot 17.2),$$
$$x_6^* = \tfrac{1}{4}(\tfrac{1}{2} \cdot 14.4 + 15.3 + 16.3 + 17.2 + \tfrac{1}{2} \cdot 18.0),$$

$p = 5$ (i.e. $k = 2$):

$$x_3^* = \tfrac{1}{5}(12.3 + 12.9 + 13.6 + 14.4 + 15.3),$$
$$x_4^* = \tfrac{1}{5}(12.9 + 13.6 + 14.4 + 15.3 + 16.3),$$
$$x_5^* = \tfrac{1}{5}(13.6 + 14.4 + 15.3 + 16.3 + 17.2),$$
$$x_6^* = \tfrac{1}{5}(14.4 + 15.3 + 16.3 + 17.2 + 18.0).$$

(In this example, there is no particular reason why we elected the length as $p = 4$ and $p = 5$ — the length was chosen arbitrarily just to have an example for the calculation.)

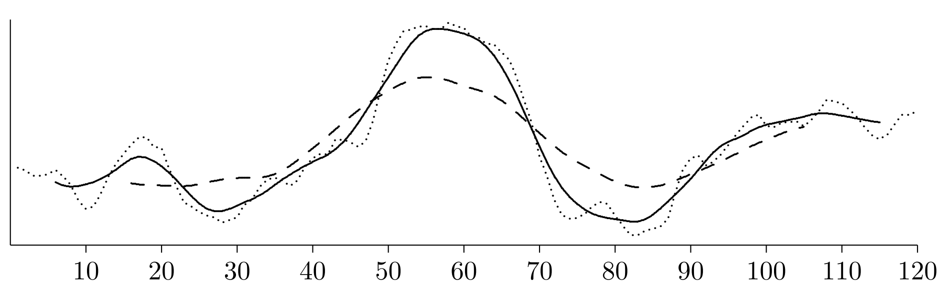

Figure 1: The influence of p on moving averages

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Programme for decomposing a given time series (x_t) with constant seasonal period p according to the additive model $x_t = g_t + s_t + r_t$:

1. Compute moving averages (x_t^*) of length p. The trend component (g_t) may be identified with these moving averages: $g_t \approx x_t^*$.

2. For $t = k+1, k+2, \ldots, T-k-1$, compute the differences

$$x_t - x_t^*.$$

   (It holds that $x_t - x_t^* \approx s_t + r_t$.)

3. For $t = k+1, k+2, \ldots, k+p$, compute the arithmetic mean $\bar{s}_t$ of the differences

$$x_t - x_t^*, \quad x_{t+p} - x_{t+p}^*, \quad x_{t+2p} - x_{t+2p}^*, \ldots$$

   (In the case of monthly data, all these differences refer to the same month; in the case of quarterly data, they refer to the same quarter.)

4. Adjust the p numbers $\bar{s}_{k+1}, \bar{s}_{k+2}, \ldots, \bar{s}_{k+p}$ such that their sum equals zero; the result may be identified with the seasonal component:

$$s_t \approx \bar{s}_t - \frac{1}{p} \sum_{i=k+1}^{k+p} \bar{s}_i.$$

5. Finally, identify the residual component with the difference:

$$r_t \approx x_t - x_t^* - s_t.$$

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Example: Capacity utilization (in percent) of food, beverages and tobacco industry in Turkey in the years 1995, 1996 and 1997. The time series[1] is given in the x_t column of the following table. The moving average length $p = 4$ (that is, $k = 2$) is appropriate since we are dealing with quarterly data. The table also contains all the necessary calculations according to the decomposition programme. The computation of all indexed numbers is explained. These computations can be carried out easily in a spreadsheet programme, such as Excel.

year	quarter	t	x_t	x_t^*	$x_t - x_t^*$	$\bar{s}_t$	s_t	r_t
1995	1st	1	68.1					
1995	2nd	2	70.6					
1995	3rd	3	73.7	[1]) 72.19	1.51	[2]) 1.43	[3])1.37	0.14
1995	4th	4	75.6	72.38	3.22	3.17	3.11	0.11
1996	1st	5	69.6	72.38	-2.78	-2.99	-3.04	0.27
1996	2nd	6	70.6	72.38	-1.78	-1.38	-1.44	-0.34
1996	3rd	7	73.7	72.36	1.34	1.43	1.37	-0.03
1996	4th	8	75.6	72.49	3.11	3.17	3.11	0.00
1997	1st	9	69.5	72.70	-3.20	-2.99	-3.04	-0.16
1997	2nd	10	71.7	72.69	-0.99	-1.38	-1.44	0.45
1997	3rd	11	74.3					
1997	4th	12	74.9					

Calculation examples:

[1]) $\frac{1}{4} \cdot (0.5 \cdot 68.1 + 70.6 + 73.7 + 75.6 + 0.5 \cdot 69.6) = 72.19$. This is simply a moving average of length $p = 4$.

[2]) $\frac{1}{2} \cdot (1.51 + 1.34) = 1.43$. This estimates $\bar{s}_t$ for the third quarter.

[3]) $1.43 - \frac{1}{4} \cdot (1.43 + 3.17 - 2.99 - 1.38) = 1.37$. We may set: $s_1 := s_5$, $s_2 := s_6$. It holds that $s_1 + s_2 + s_3 + s_4 = 0$.

The seasonal component is highest in the fourth quarter and lowest in the first quarter of a year: In the fourth quarter, capacity utilization in this branch is about three percent points above average. This is plausible since a year's harvest is processed mainly in the fourth quarter, which increases the food processing factories' capacity utilization. — We omit a graphical representation of this modest example...

[1]Source: Table 227 in DİE: *İstatistik Yıllığı 1998*