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FM 431: Econometrics of Financial Markets

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ACF and PACF of ARMA Processes

An ARMA(p, q) process $(y_t)_t$ is given by

$$y_t = a_0 + a_1 y_{t-1} + \cdots + a_p y_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \cdots + \beta_q \varepsilon_{t-q}$$

(with suitable properties of the lag polynomials), where $(\varepsilon_t)_t$ is white noise.

Model	ACF	PACF
AR(1)	$\rho_s = a_1^s$ (dies down in a damped exponential fashion)	(cuts off after lag 1)
AR(2)	$\rho_1 = \frac{a_1}{1-a_2}$ $\rho_2 = \frac{a_1^2}{1-a_2} + a_2$ $\rho_s = a_1\rho_{s-1} + a_2\rho_{s-2}$ for $s \geq 3$ (dies down according to a mixture of damped exponentials and/or damped sine waves)	(cuts off after lag 2)
MA(1)	$\rho_1 = \frac{\beta_1}{1+\beta_1^2}$ $\rho_s = 0$ for $s \geq 2$ (cuts off after lag 1)	(dies down in a fashion dominated by damped exponential decay)
MA(2)	$\rho_1 = \frac{\beta_1(1+\beta_1)}{1+\beta_1^2+\beta_2^2}$ $\rho_2 = \frac{\beta_2}{1+\beta_1^2+\beta_2^2}$ $\rho_s = 0$ for $s \geq 3$ (cuts off after lag 2)	(dies down according to a mixture of damped exponentials and/or damped sine waves)
ARMA(1,1)	$\rho_1 = \frac{(1+a_1\beta_1)(a_1+\beta_1)}{1+\beta_1^2+2\beta_1 a_1}$ $\rho_s = a_1\rho_{s-1}$ for $s \geq 2$ (dies down in a damped exponential fashion)	(dies down in a fashion dominated by damped exponential decay)