FEC 522: Financial Econometrics II

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İSTANBUL BİLGİ ÜNİVERSİTESİ

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- R files used for this course are available upon request.

Chapter 7:

Bivariate GARCH Models

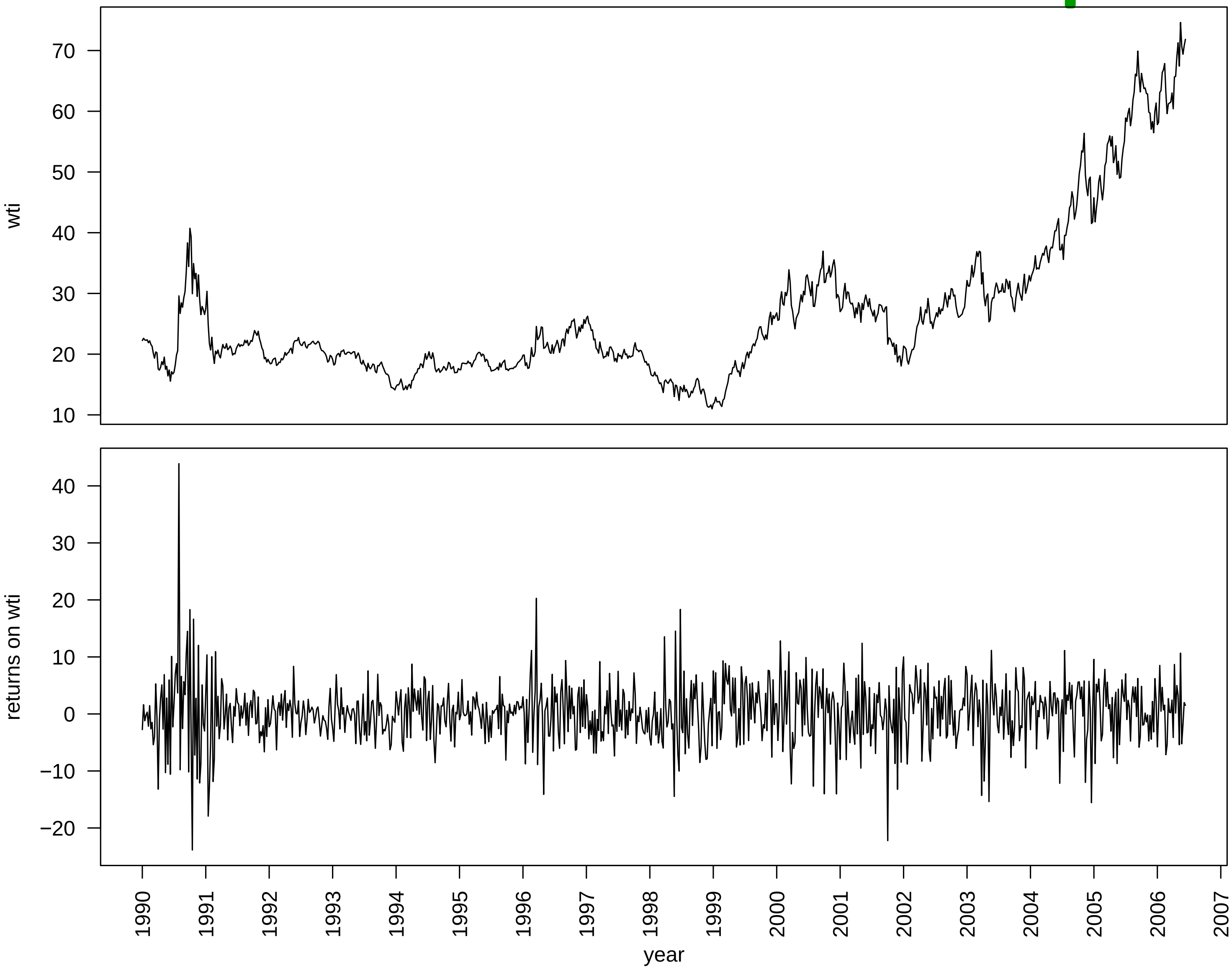
7.1 Introduction: An Example

- The stock market and the crude oil market are "somehow related".

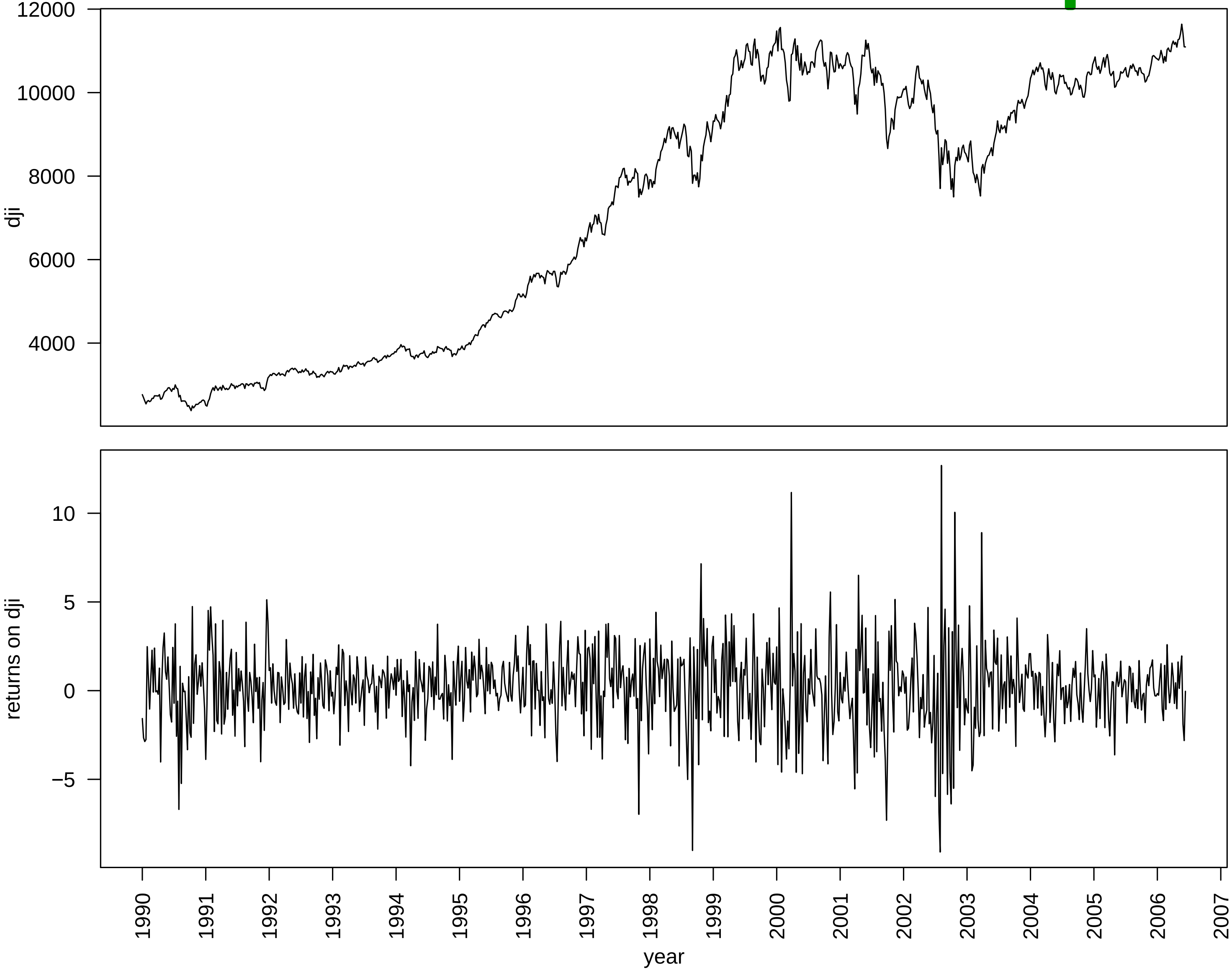
- Project aim: Cast some light into the interaction of the crude oil market and the stock market.

- As an example, we use West Texas Intermediate (wti) crude oil and the Dow-Jones stock index (dji).

7.1 Introduction: An Example



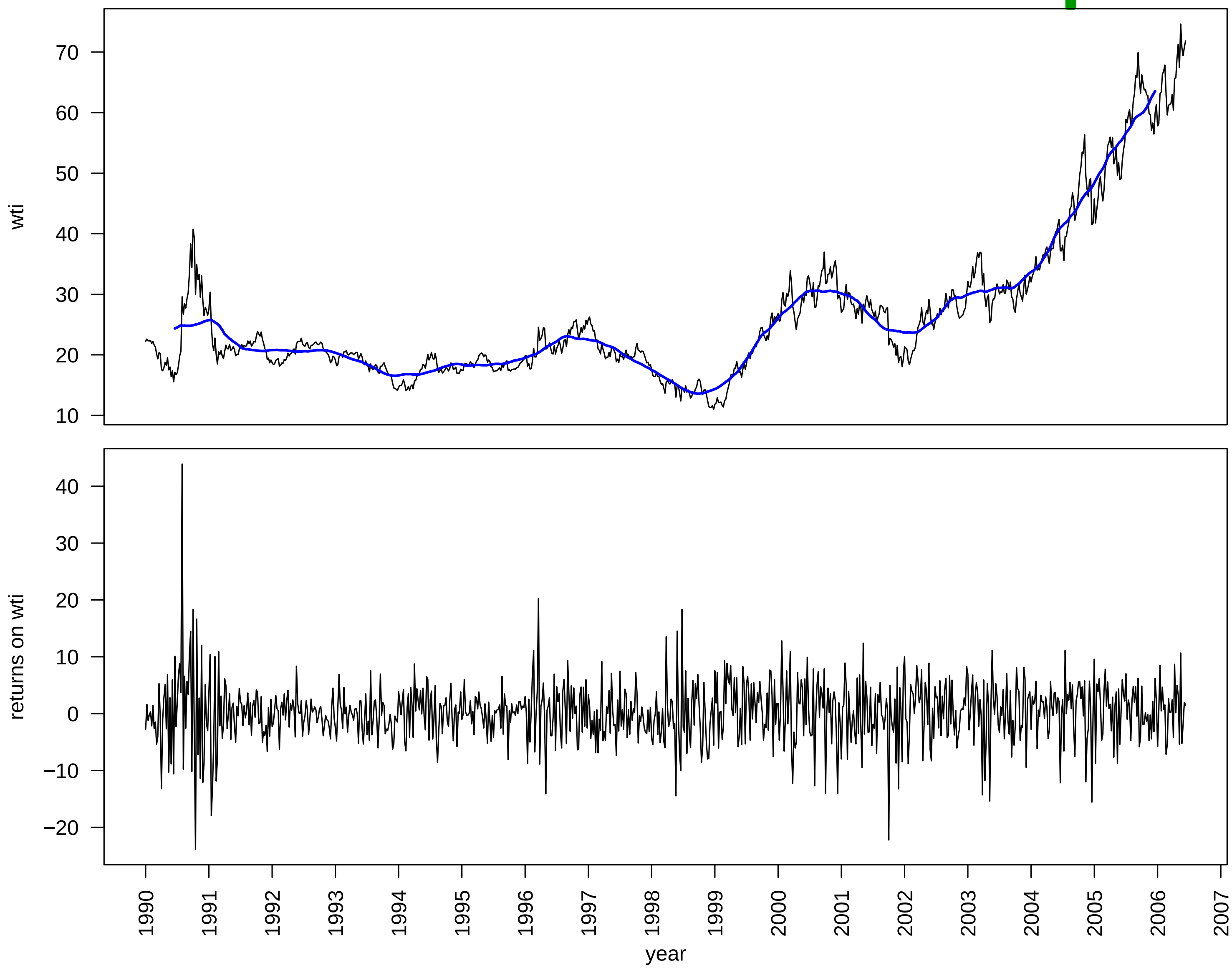
7.1 Introduction: An Example



7.1 Introduction: An Example

	dji	wti
first day	1990-01-09	1990-01-09
last day	2006-05-30	2006-05-30
observations	856	856
mean	0.18413	0.27214
std error	0.06935	0.20060
var	4.73157	27.86522
std deviation	2.17522	5.27875
skewness	0.10808	0.39352
std error	0.29678	0.57873
kurtosis	3.34601	6.46199
std error	0.97727	3.64780

7.1 Introduction: An Example



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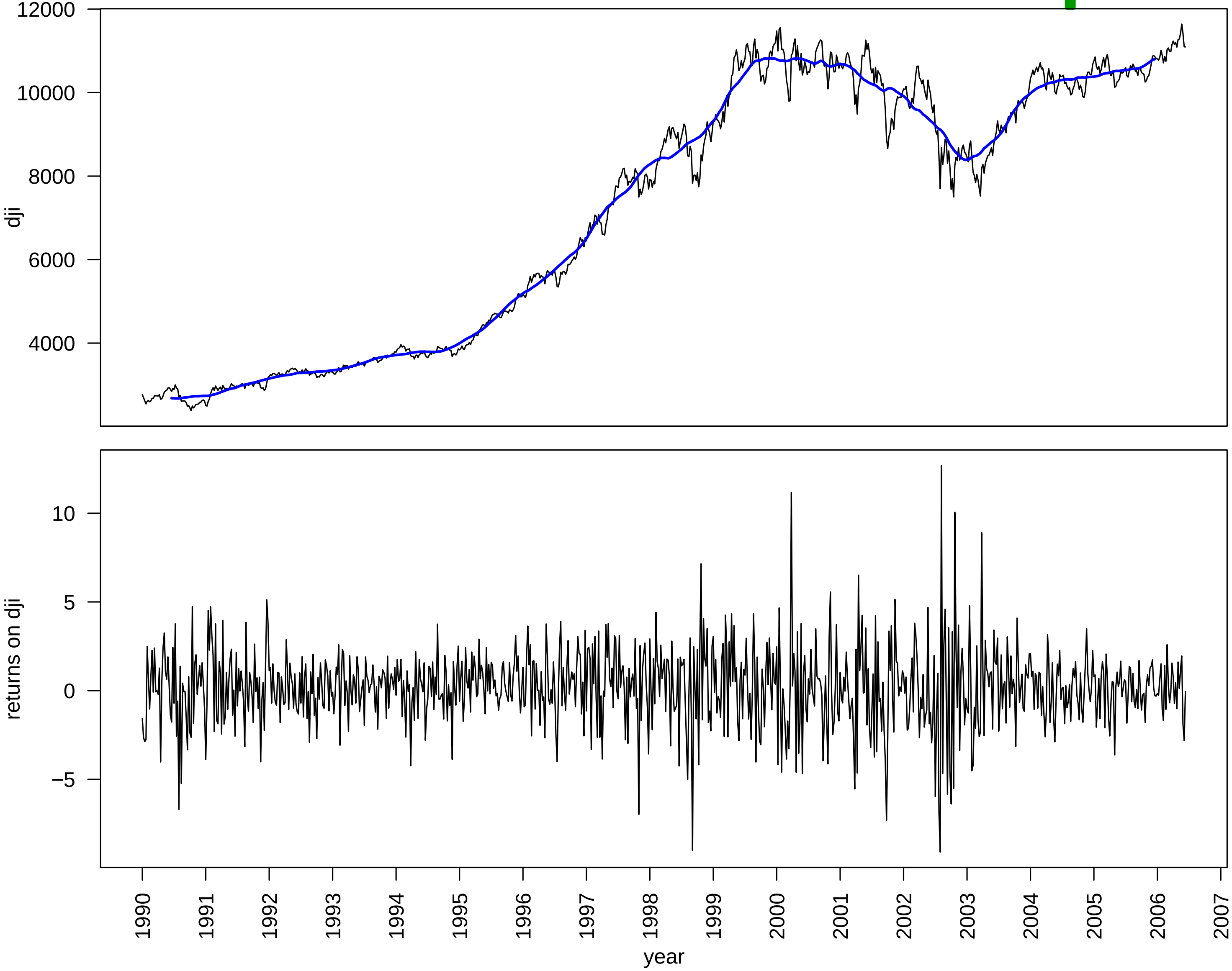


7.1 Introduction: An Example

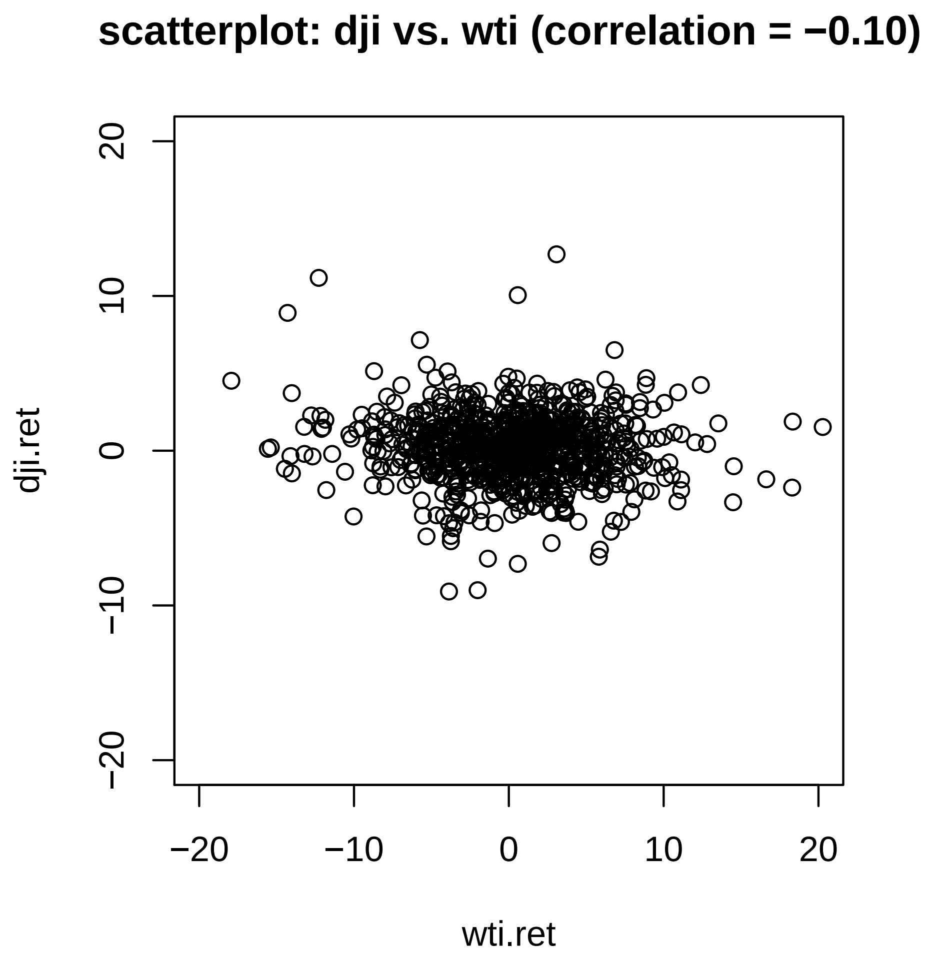
The trend-specific **average conditional volatility** is:

	wti	dji
up	5.28	2.20
down	4.97	2.33

This means:

- There is evidence of asymmetry in news impact on volatility, in wti as well as in dji.
- Asymmetry is in opposite directions.

7.1 Introduction: An Example



7.1 Introduction: An Example

The trend-specific average conditional volatility of dji is:

		dji trend	
		up	down
wti trend	up	2.20	2.38
	down	2.21	2.25

This means:

- There is also evidence of joint asymmetry in news impact on volatility.

7.1 Introduction: An Example

The analysis so far is insufficient:

- We looked only at averages.
- The GARCH model used does not allow for asymmetry.
- The models are univariate.

7.2 News Impact in GARCH Models

The univariate GARCH model.

$$X_t = \mu_t + \epsilon_t, \quad \epsilon_t = \sqrt{h_t} \cdot \nu_t,$$

where $(\nu_t)_t$ is white noise with $\text{var}(\nu_t) = 1$. Then:

$$\text{var}(\epsilon_t | \mathcal{F}_{t-1}) = h_t.$$

How is h_t defined?

7.2 News Impact in GARCH Models

Definition of h_t.

- $h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta h_{t-1}$
(GARCH; Engle 1982, Bollerslev 1986)

- $h_t = h_{t-1}^{\beta} \exp\left(\alpha - \gamma\sqrt{2/\pi}\right) \exp\left[\left(\theta + \mathsf{sign}(\epsilon_{t-1}) \cdot \gamma\right) \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}\right]$
(EGARCH; Nelson 1990)

- $h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta h_{t-1} + \gamma \cdot S \cdot \epsilon_{t-1}^2$
(Glosten, Jaganathan, Runkle 1989)

The function $\epsilon_{t-1} \mapsto h_t$ is called the news impact function (Engle, Ng 1993)

7.3. News Impact in MGARCH Models

The bivariate case.

$$X_t = M_t + \epsilon_t, \quad \epsilon_t = \mathtt{H}_t^{1/2} \cdot \nu_t$$

Then, the conditional covariance matrix of ϵ_t is

$$\text{cov}(\epsilon_t | \mathcal{F}_{t-1}) = \mathtt{H}_t,$$

and we can study news impact on $\mathtt{H}_t$ in terms of:

- h_{11t}, the conditional variance of ϵ_{1t},
- h_{22t}, the conditional variance of ϵ_{2t},
- h_{12t}, the conditional covariance of ϵ_{1t} and ϵ_{2t},
- $h_{12t}/\sqrt{h_{11t} \cdot h_{22t}}$, the conditional correlation of ϵ_{1t} and ϵ_{2t},
- $\det(\mathtt{H}_t)$, the determinant of $\mathtt{H}_t$.

How can $\mathtt{H}_t$ be specified?

7.3. News Impact in MGARCH Models

The MGARCH-BEKK model.

The conditional covariance matrix is defined as

$$\mathtt{H}_t = \mathtt{C}'\mathtt{C} + \underbrace{\mathtt{A}'\epsilon_{t-1}\epsilon_{t-1}'\mathtt{A}}_{\text{ARCH term}} + \underbrace{\mathtt{B}'\mathtt{H}_{t-1}\mathtt{B}}_{\text{GARCH term}}$$

with parameter matrices

$$\mathtt{C} = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}, \quad \mathtt{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \mathtt{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

7.3 News Impact in MGARCH Models

The DCC-GARCH model (Tse & Tsui).

The conditional covariance matrix is defined as

$$\mathtt{H}_t = \mathtt{D}_t \mathtt{R}_t \mathtt{D}_t,$$

where

$$\begin{array}{lcl} \mathtt{D}_t & = & \text{diag}(h_{it}) \quad (h_{it}\text{: from univariate GARCH processes}) \\ \mathtt{R}_t & = & (1-\theta_1-\theta_2)\mathtt{R} + \theta_1 \Psi_{t-1} + \theta_2 \mathtt{R}_{t-1} \\ \Psi_{t-1} & = & \text{sample correlation matrix, past } M \text{ time periods} \\ \mathtt{R} & = & \text{a correlation matrix} \end{array}$$

and $\theta_1, \theta_2 \geq 0$, $\theta_1 + \theta_2 < 1$.

7.3. News Impact in MGARCH Models

The bivariate asymmetric quadratic GARCH model.

The conditional covariance matrix is defined as

$$
\begin{aligned}
\mathtt{H}_t \quad = \quad & \mathtt{C}'\mathtt{C} \quad + \quad \mathtt{A}'\epsilon_{t-1}\epsilon'_{t-1}\mathtt{A} \; + \; \mathtt{B}'\mathtt{H}_{t-1}\mathtt{B} \\
& + S_w(\epsilon_{t-1}) \cdot \Gamma'\epsilon_{t-1}\epsilon'_{t-1}\Gamma
\end{aligned}
$$

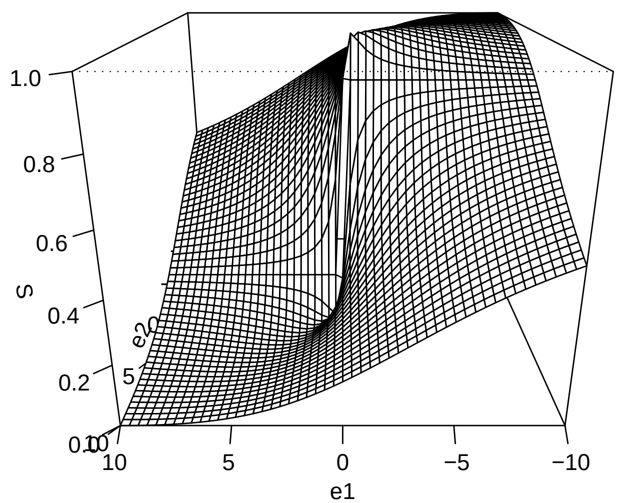
with the additional parameter matrix

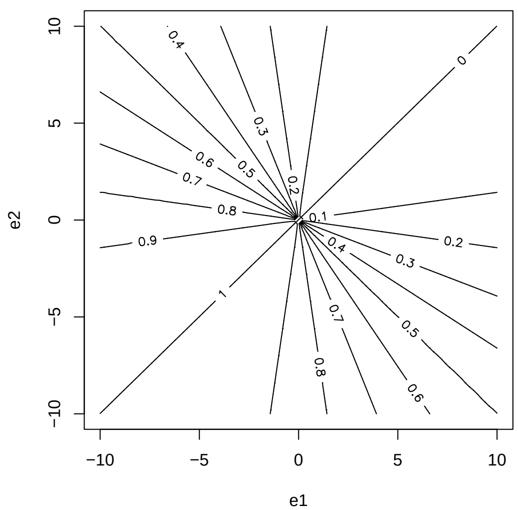
$$
\Gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}
$$

and a weight function S_w.

7.3. News Impact in MGARCH Models

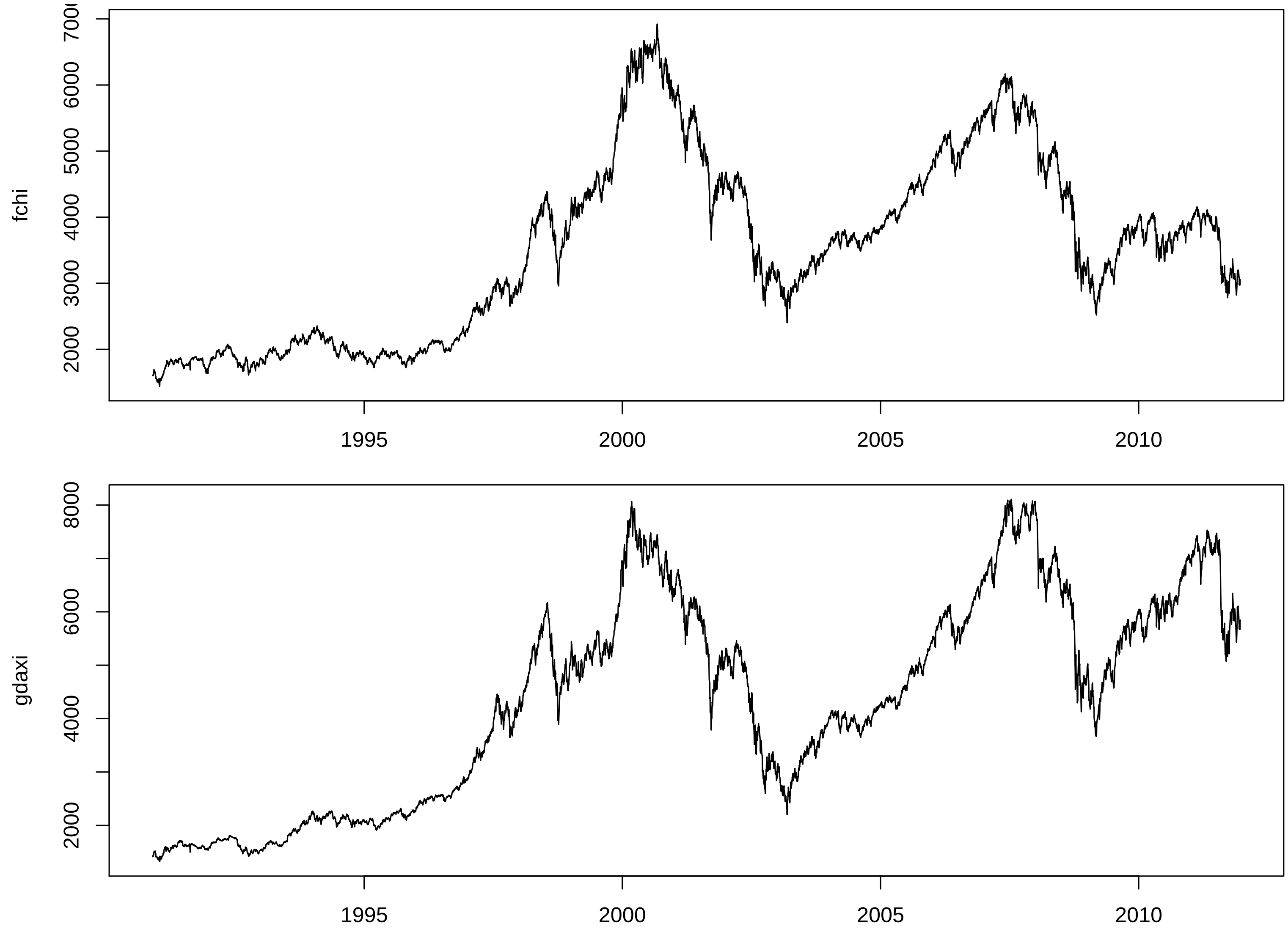
The weight function S_w.





$$S_w(e_1, e_2) = 0.5 - \frac{\cos(\frac{\pi}{4} + w) \cdot e_1 + \sin(\frac{\pi}{4} + w) \cdot e_2}{2\sqrt{e_1^2 + e_2^2}}$$

7.4 Example: CAC40 and DAX



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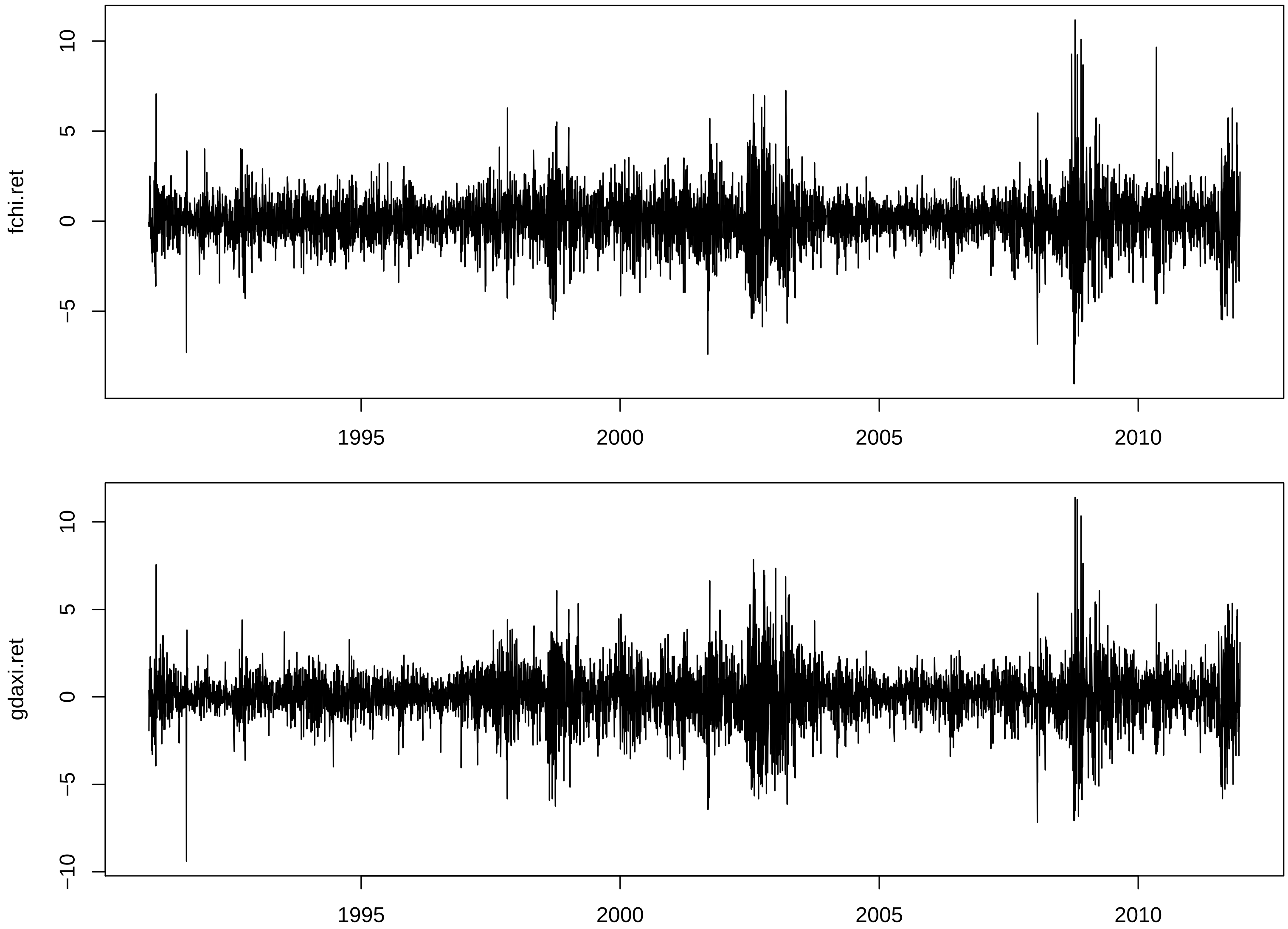

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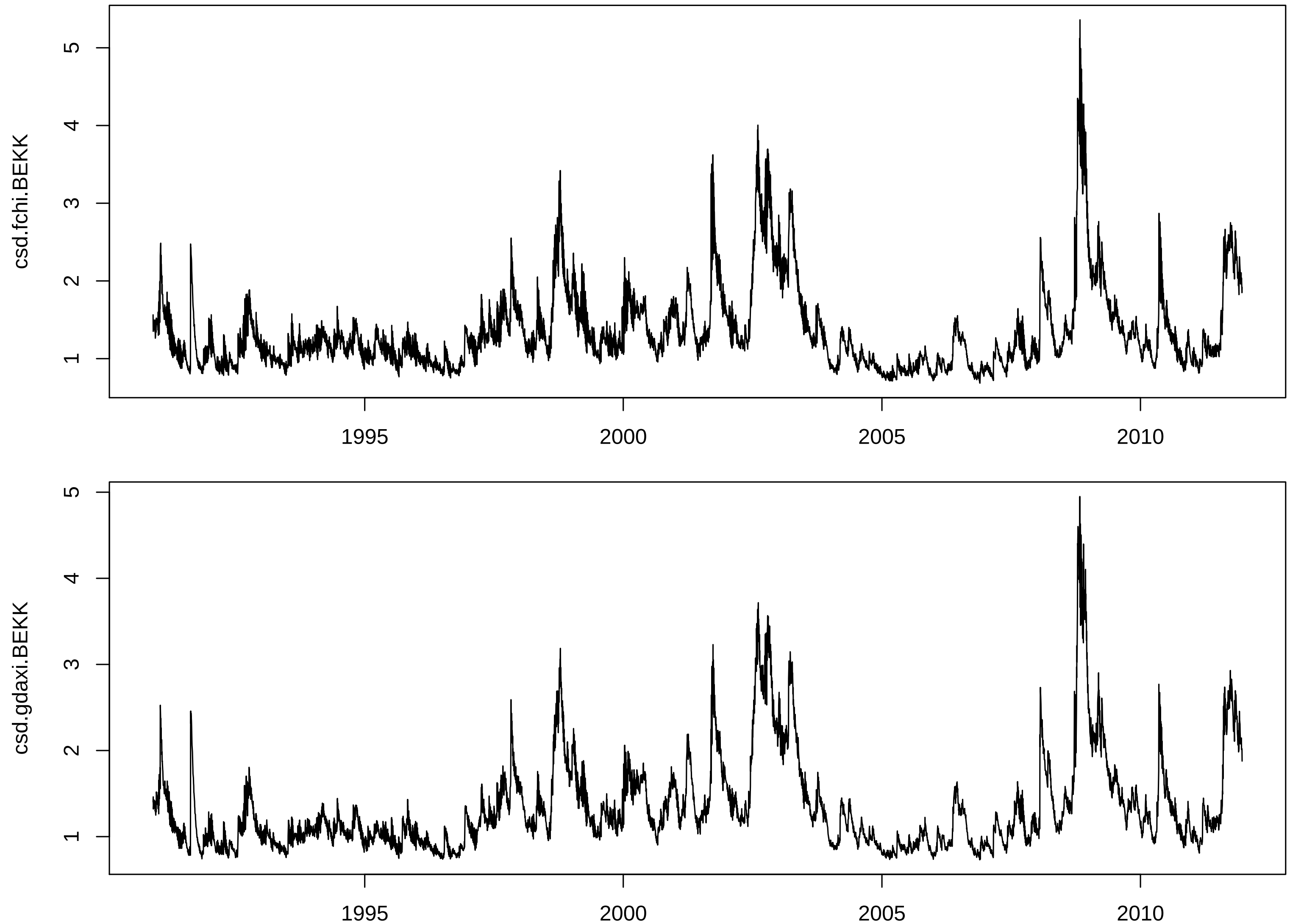

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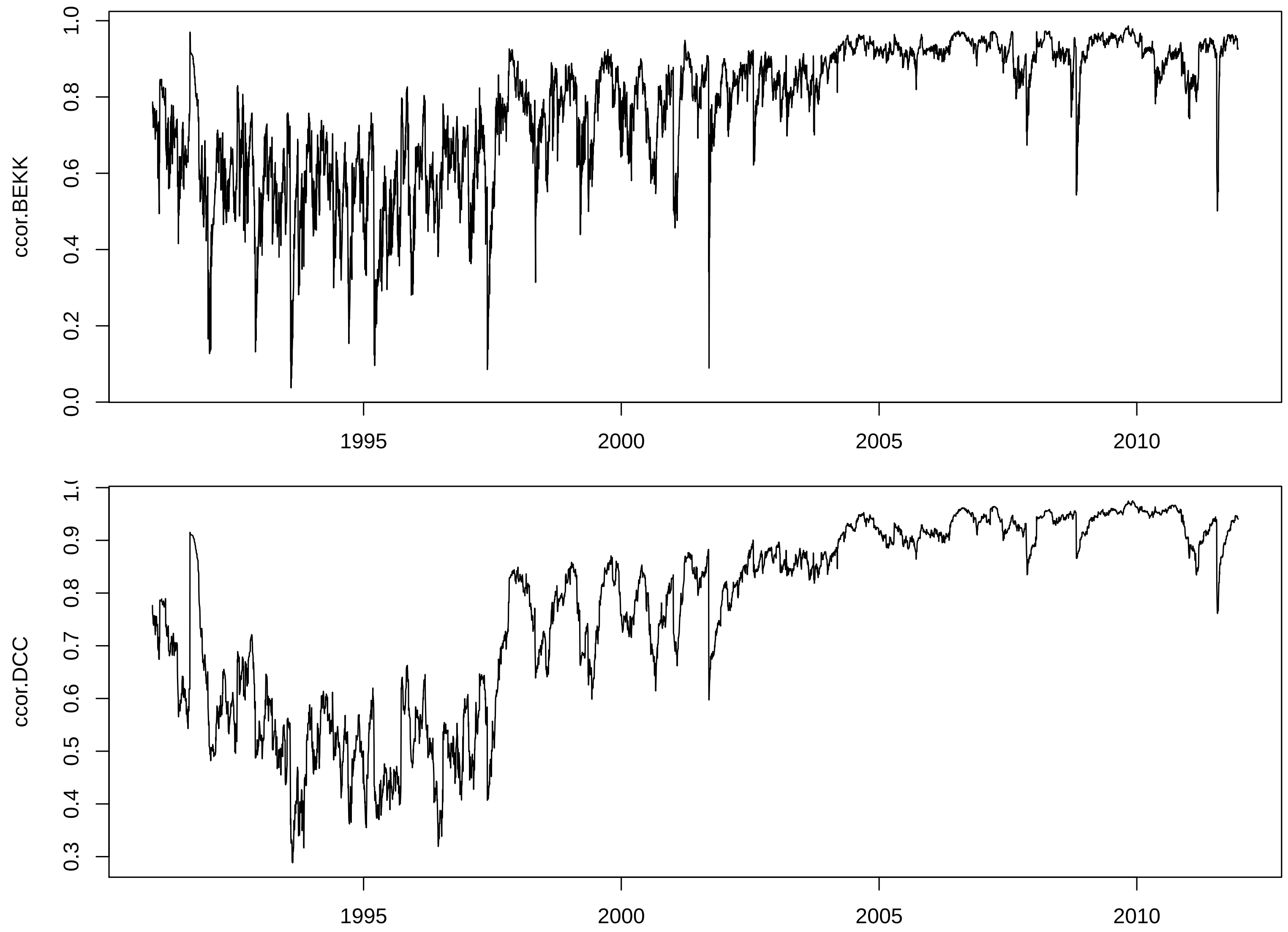

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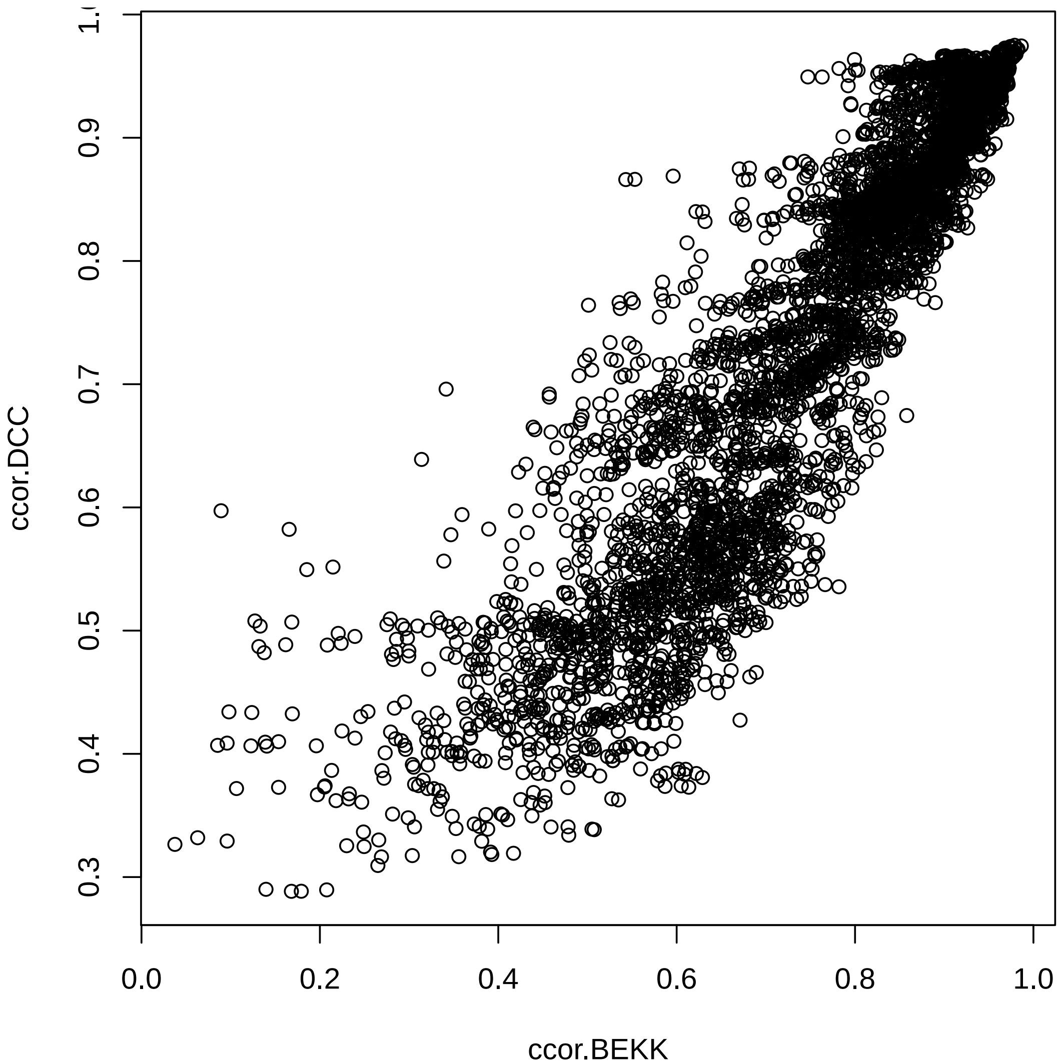


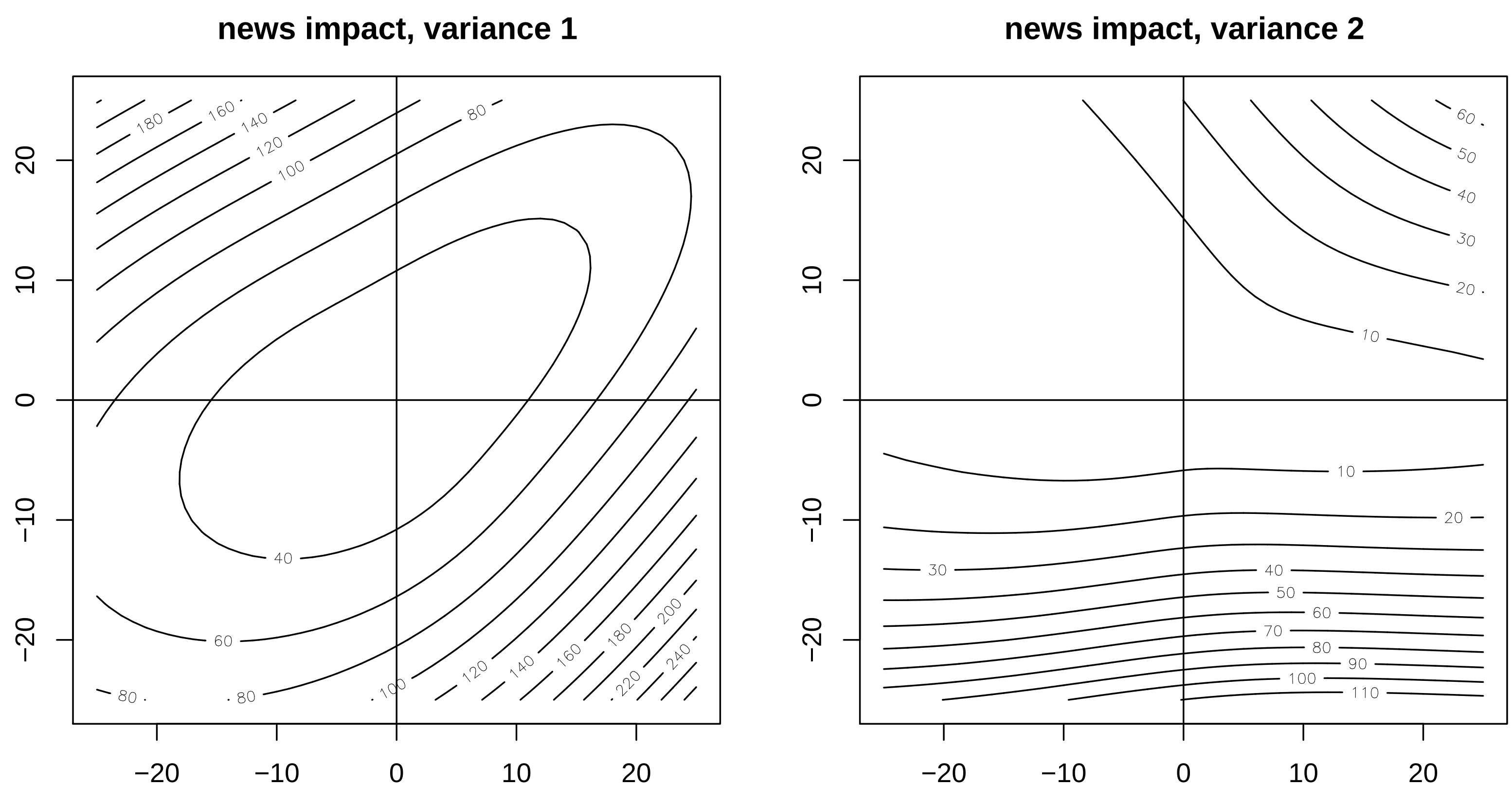
7.5 Example: WTI and DJI

In what follows, we shall:

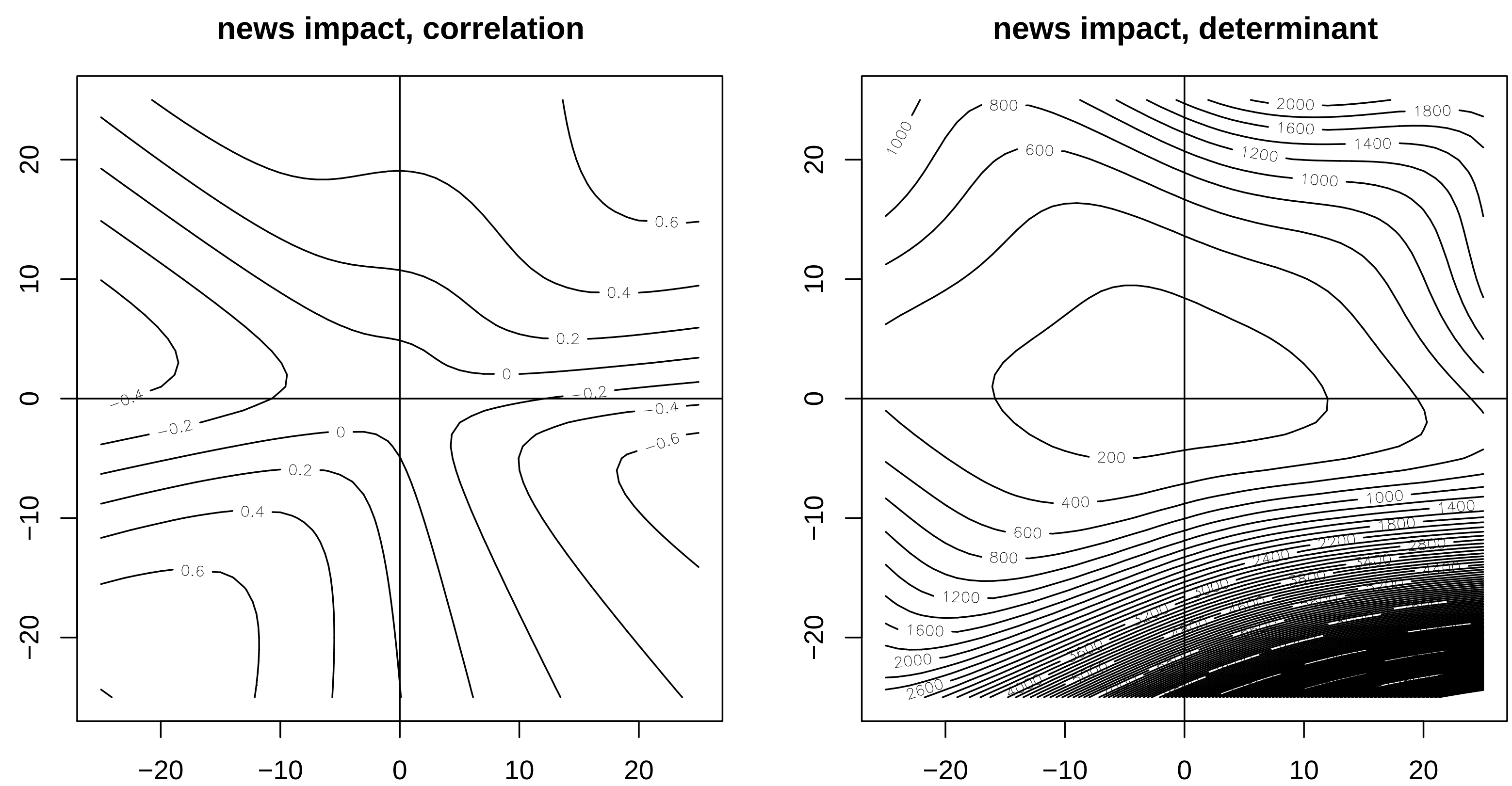
- Fit the baq GARCH model to the bivariate series consisting of wti and dji.
- Inspect the news impact on the variances, correlation, and the determinant.

All considered models produce white noise residuals.

7.5 Example: WTI and DJI



7.5 Example: WTI and DJI



7.5 Example: WTI and DJI

The estimated parameters are:

$$C = \begin{pmatrix} \underset{(0.152)}{1.095} & \underset{(-)}{0} \\ \underset{(-)}{0} & \underset{(0.119)}{0.236} \end{pmatrix} \qquad A = \begin{pmatrix} \underset{(0.047)}{0.199} & \underset{(0.014)}{-0.069} \\ \underset{(0.086)}{-0.362} & \underset{(0.046)}{-0.101} \end{pmatrix}$$

$$B = \begin{pmatrix} \underset{(0.015)}{0.913} & \underset{(0.005)}{0.019} \\ \underset{(0.029)}{0.091} & \underset{(0.016)}{0.934} \end{pmatrix} \qquad \Gamma = \begin{pmatrix} \underset{(0.057)}{0.335} & \underset{(-)}{0} \\ \underset{(-)}{0} & \underset{(0.056)}{0.418} \end{pmatrix}$$

$$w = \underset{(0.207)}{1.38}$$

7.5 Example: WTI and DJI

Conclusions.

- The interplay between the crude oil market and the stock market manifests itself in conditional second moments, rather than in correlation of returns.
- This interplay is asymmetric.
- Our findings can be used in risk management and portfolio construction.