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# FEC 522: Financial Econometrics II

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## Some problems

**Problem 1:** Using the normal distribution in the context of finance and econometrics was heavily criticized by some scholars, notably Nassim Nicholas Taleb.

a) Explan briefly the reasons brought forward against using the normal distribution.

b) When you think of your term project: Where was the normal distribution involved? You need not write all model equations and assumptions explicitly, but give a precise explanation in your own words.

c) Considering (b): Is there an alternative to using the normal distribution? Do you think that would be a good idea? Discuss briefly.

**Problem 2:** A GARCH model was fitted to a series of 1000 observations, starting 2009-06-11 and ending 2013-05-31, of daily returns in percent on the stock index XU100 (Istanbul Stock Exchange). The result is:

$$X_t = 0.102 + \epsilon_t, \quad \epsilon_t = \nu_t \cdot \sqrt{h_t}, \quad h_t = 0.146 + 0.1068\epsilon_{t-1}^2 + 0.8212h_{t-1}.$$

Here, $X_t$ designates the return on day $t$ and $(\nu_t)$ is Gaussian white noise.
The return on day $t = 1000$ (2013-05-31) was: $-1.354$, and $h_{1000}$ turned out to be 2.27. For day $t + 1 = 1001$ (2013-06-03, that is, last Monday), a return of $x_{1001} = -10.47$ was observed, and the variance forecast for that day was 2.24.

a) Explain how $h_{1001}$ is computed. (Write the exact mathematical expression, with all values plugged in correctly.)

b) Determine the distribution of $X_{1001}$.

c) In the light of (b), is the actually observed return on day $t = 1001$ a surprise, or could we have expected such a big loss? Give reasons for your answer. (Hint: $\sqrt{2.24} \approx 1.5$.)

d) Using this example, explain why GARCH is a conditional variance model.

e) Which information do the model equations give us about the *unconditional* variance of daily returns?

f) Comment on the outcome in (c) from the point of view established in Problem 1.

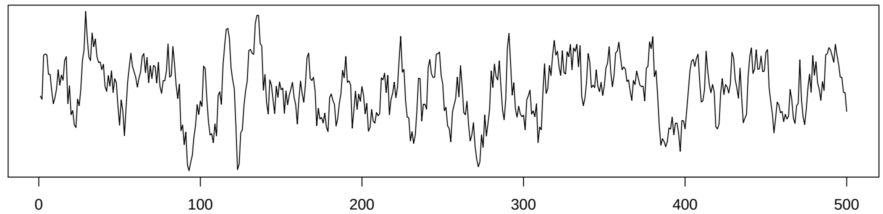
**Problem 3:** Let a stochastic process $(X_t)$ be given as follows: $X_t = 2 + 0.5X_{t-1} + \epsilon_t$, where $(\epsilon_t)$ is white noise with $\text{var}(\epsilon_t) = 3$.

a) Explain briefly why this is a "conditional expectation" model, but not a "conditional variance" model.

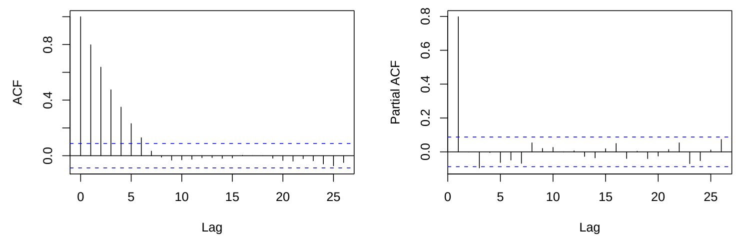
b) Suppose we observed $X_t = 2.5$, $X_{t-1} = 1.7$. Compute a forecast for $X_{t+1}$.

c) Suppose we observed $X_t = 2.5$, $X_{t-1} = 1.7$. Compute a forecast for the variance of the process at time $t + 1$, that is, for the variance of $X_{t+1}$.

**Problem 4:** The following figure shows a simulated sample path of a stochastic processes:



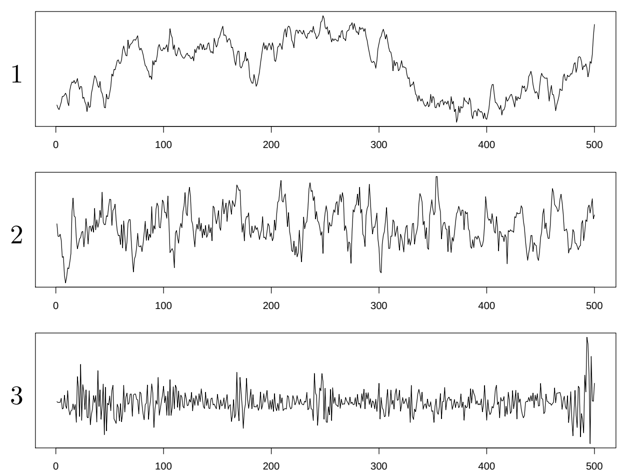

The empirical autocorrelation function and the empirical partial autocorrelation functions are:




a) Which stochastic model might have been used to simulate the series? Give reasons for your answer.

b) Explain briefly what it means when we say that the series was created on the PC by simulation. (What do we need? How do we proceed for the simulation? Which components do we have to define? — etc.)

**Problem 5:** The following three figures show simulated sample paths of stochastic processes:




Which of these processes could be a simulated AR(1) process? (Give reasons for your answer.)

**Problem 6:** The following R code simulates a stochastic process:

```
eps = rnorm(500)
x = 0
for (t in 2:500) { x[t] = 0.7 * x[t-1] + eps[t] }
```

Explain which process is simulated. (Write down the equations which define this process.)