Market connectedness: spillovers, information flow, and relative market entropy

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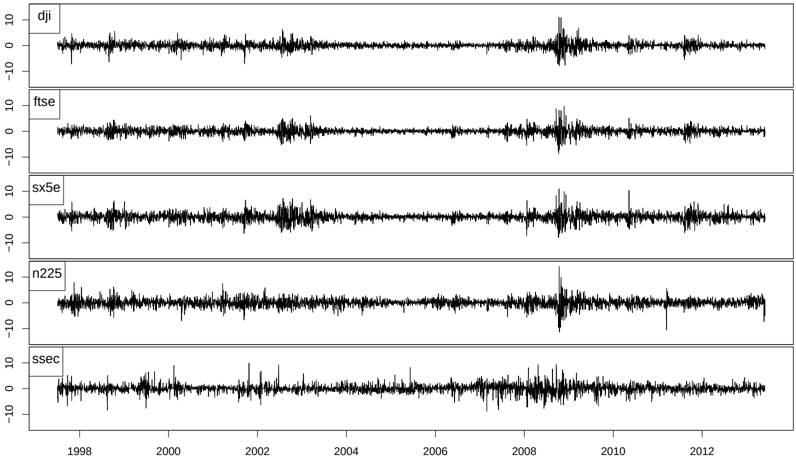
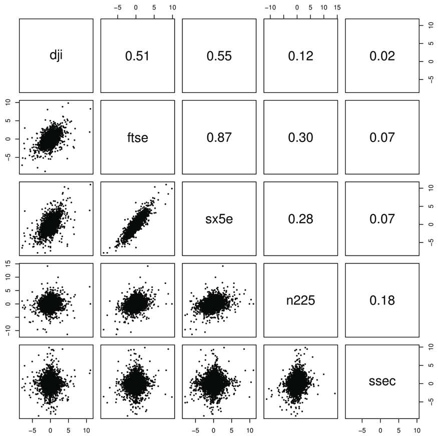
Market connectedness and fevd

- Assessing the degree of connectedness of equity markets
- Diebold & Yilmaz, 2005–2014:
  - VAR model for return series
  - forecast error variance decomposition (fevd)
  - spillover table
  - collapsed into the spillover index
- Our contribution:
  - characterization of dynamic behavior
  - a Markov chain perspective
  - application of entropy measures

The series of daily returns



Scatterplots of daily returns



Forecast error variance decomposition — four markets A, B, C, D.

- Forecast error variance (market A; Φ = an irf):

$$\mathrm{var}\left\{ \sum_{i=0}^{n-1} \left(\Phi_A^A(i),\ \Phi_A^B(i),\ \Phi_A^C(i),\ \Phi_A^D(i) \right) \times \begin{pmatrix} \epsilon_{A,t+n-i} \\ \epsilon_{B,t+n-i} \\ \epsilon_{C,t+n-i} \\ \epsilon_{D,t+n-i} \end{pmatrix} \right\}$$

- This expression equals

$$\sigma_A^2 \cdot \sum_{i=0}^{n-1} (\Phi_A^A)^2(i) \quad + \quad \sigma_B^2 \cdot \sum_{i=0}^{n-1} (\Phi_A^B)^2(i) \quad +$$
$$\sigma_C^2 \cdot \sum_{i=0}^{n-1} (\Phi_A^C)^2(i) \quad + \quad \sigma_D^2 \cdot \sum_{i=0}^{n-1} (\Phi_A^D)^2(i)$$

Spillover table & spillover index

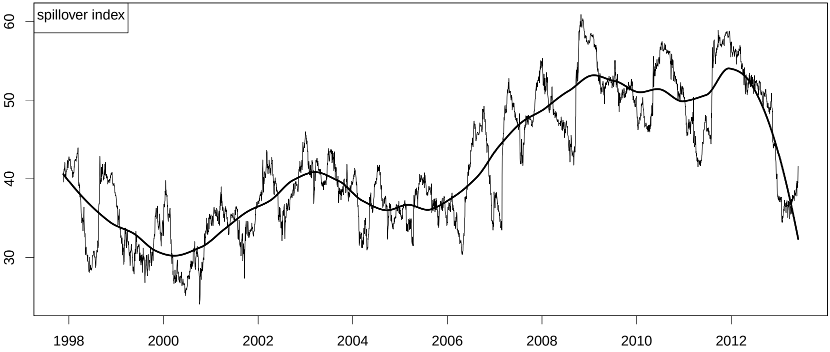
- Spillover table, schematically:

		from (time t)			
		A	B	C	D
to (time $t+n$)	A	$\square$	$\blacksquare$	$\blacksquare$	$\blacksquare$
	B	$\blacksquare$	$\square$	$\blacksquare$	$\blacksquare$
	C	$\blacksquare$	$\blacksquare$	$\square$	$\blacksquare$
	D	$\blacksquare$	$\blacksquare$	$\blacksquare$	$\square$

- Spillover index $= \dfrac{\sum \blacksquare}{\sum \blacksquare + \sum \square}$ (Diebold & Yilmaz, 2005)

Example: dji, ftse, sx5e, n225, ssec

Spillover index series:



The need for summary

- For each day: the procedure yields an $n \times n$ table.
- Spillover index: summary measure.
- If spillover index $= 40\%$, what is the spillover table? (3 markets)

$$\underset{\text{I}}{\overset{\dots\text{this?}}{\begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}}} \qquad \underset{\text{II}}{\overset{\dots\text{or this?}}{\begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}}}$$

- "Average" spillover of shocks to other markets: 40%!

A hypothetical shock hitting the network

- Spillover matrix:
  - most recent information available for a day
  - defines weights in a network
- Hypothetical shock to node (or market) i on day t:

$$\mathbf{n}_0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{array}{l} \\ \\ \\ \leftarrow i\text{-th component} \\ \\ \\ \\ \end{array}$$

- What happens when such a shock hits the network?

The size of a shock

- Assumptions
  - Given: A spillover matrix $\mathbf{M}_t$ for day t
  - Propagation of a shock within next day:
    - initial shock size: $\mathbf{n}_0$ (a unit vector)
    - shock propagation in short time interval according to

$$\mathbf{n}_{s+1} = \mathbf{M}_t \cdot \mathbf{n}_s, \quad s = 0, 1, \ldots$$

- Question: What happens to shock size $\mathbf{n}_s$ as $s \to \infty$?

The size of a shock

- It holds that:

  The relative size of a shock, as $s \to \infty$, is determined by the left eigenvector of the spillover matrix.

- left eigenvector: value of a shock to which the market is exposed as seed for future variability or risk ("propagation value")
- Example:

$$\underset{\text{I}}{(1,2,2)\begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}} \qquad \underset{\text{II}}{(1,0.30,0.26)\begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}}$$

The location of a shock

- Can we use the spillover table as a Markov transition matrix?
- $\mathbf{M}_t$ is row-stochastic: If p (column vector) is a probability distribution, then:
    - $\mathbf{M}_t \cdot p$ need not be a distribution
    - $p' \cdot \mathbf{M}_t$ is a distribution
- However, a Markov chain with $p'_{s+1} = p'_s \cdot \mathbf{M}_t$ is running backward in time (relative to the setup of $\mathbf{M}_t$).
- p: distribution of shock location; $p' \cdot \mathbf{M}_t$: distribution of shock origin
- Time needs to be reversed.

The location of a shock: time reversal

- A Markov chain running forward in time can be defined for strongly connected networks.
- Transformation of $\mathbf{M}_t$ into a forward Markov transition matrix (using the eigenvalue structure):

$$\mathbf{V}_t^{-1} \cdot \mathbf{M}_t' \cdot \mathbf{V}_t$$

(Tuljapurkar, 1982)

- A Markov chain with $p_{s+1}' = p_s' \cdot \mathbf{V}_t^{-1} \cdot \mathbf{M}_t' \cdot \mathbf{V}_t$ is running forward in time.

The location of a shock: distributional characteristics

- If a shock hits a node (market, asset) of the network on day t:
  - Where will the (hypothetical) shock be settling?
  - Share of time spent in each node of the network?
  - Stationary distribution of the Markov chain?
- It can be shown that:

  The stationary probability distribution of the Markov chain running forward in time equals the (normed) left eigenvector of $\mathbf{M}_t$.

- Dual interpretation of propagation values!

Markov chain interpretation: two perspectives

- Single-day perspective:
  - Information flow according to a single $\mathbf{M}_t$?
  - Emphasizes what happens on a day in isolation.
  - Sensitive to *any* change between days.
- Day-to-day perspective:
  - Shock in the (remote) past.
  - $(\mathbf{M}_t)_{t=0,1,\ldots}$ defines a non-homogeneous Markov chain.
  - Emphasizes what happens from day to day.
  - Sensitive to *abrupt* changes only.

Relative market entropy

- Several probability distributions are associated with each day:
  - initial shock distribution (a unit vector)
  - stationary distribution of shock position
  - non-stationary distribution of shock position
- Informational distance between distributions?
- Relative market entropy: Kullback-Leibler distance, defined as

$$\text{KLIC} \;=\; \sum_i \pi_a(i) \cdot \log_2 \left(\frac{\pi_a(i)}{\pi_b(i)} \right)$$

with (for example)

$$\begin{aligned} \pi_a &= \text{today's (non-) stationary distribution,} \\ \pi_b &= \text{yesterday's (non-) stationary distribution} \end{aligned}$$

Entropy & KLIC

- Given:
  - random variable X with distribution $\mathbb{P}$
  - random variable Y with distribution $\mathbb{Q}$
- Entropy of X:

$$H(X) = -\sum_{x} \mathbb{P}(x) \cdot \log_2 \mathbb{P}(x)$$

- Kullback-Leibler divergence (KLIC) of (false) $\mathbb{Q}$ from (true) $\mathbb{P}$:

$$D_{KL}(\mathbb{P} \parallel \mathbb{Q}) = \sum_{x} \mathbb{P}(x) \cdot \log_2 \frac{\mathbb{P}(x)}{\mathbb{Q}(x)}$$

Example: entropy

- Given: random variable X and its distribution $\mathbb{P}$

	A	B	C	D
$X, \mathbb{P}$	$1/2$	$1/4$	$1/8$	$1/8$

- Suppose you know the support $\{A, B, C, D\}$ and $\mathbb{P}$.
- Average number of ("clever") guesses required to identify a realization:

$$1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8} = 1.75$$

- This equals the entropy of X:

$$H(X) = -\frac{1}{2} \cdot \log_2 \frac{1}{2} - \frac{1}{4} \cdot \log_2 \frac{1}{4} - \frac{1}{8} \cdot \log_2 \frac{1}{8} - \frac{1}{8} \cdot \log_2 \frac{1}{8}$$

- Case of a uniform distribution: $H(X) = 2$.

Example: KLIC

- Given: random variables X, Y, their respective distributions $\mathbb{P}$, $\mathbb{Q}$

	A	B	C	D
X, $\mathbb{P}$	1/2	1/4	1/8	1/8
Y, $\mathbb{Q}$	1/8	1/8	1/4	1/2

- Suppose you know the support $\{A, B, C, D\}$ and $\mathbb{Q}$, but not $\mathbb{P}$.
- Average number of guesses required to identify a realization of X when using the coding for Y:

$$1 \cdot \frac{1}{8} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{4} + 3 \cdot \frac{1}{2} = 2.625 > 1.75$$

- The difference equals the Kullback-Leibler divergence of (false) $\mathbb{Q}$ from (true) $\mathbb{P}$:

$$D_{KL}(\mathbb{P} \parallel \mathbb{Q}) = \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 1 - \frac{1}{8} \cdot 1 - \frac{1}{8} \cdot 2 = 0.875$$

Kolmogorov-Sinai entropy

- Idea: The system will converge to equilibrium after being hit by a shock.
- How fast will it converge?
- Appropriate measure: Kolmogorov-Sinai entropy,

$$\mathrm{KS} = -\sum_{i,j} \pi(i) \cdot \log_2 \left(\mathbf{p}_{ij}^{\mathbf{p}_{ij}} \right),$$

  where

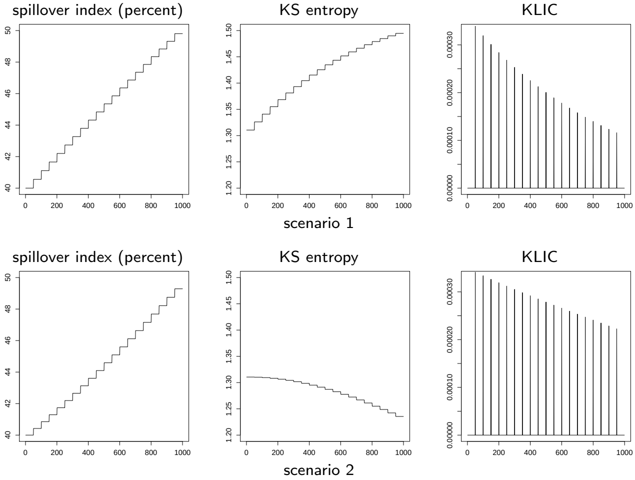
$$\begin{aligned} \pi &= \text{stationary distribution}, \\ \mathbf{p}_{ij} &= \text{entries of the transition matrix}. \end{aligned}$$

- Can be seen as a measure of network stability.

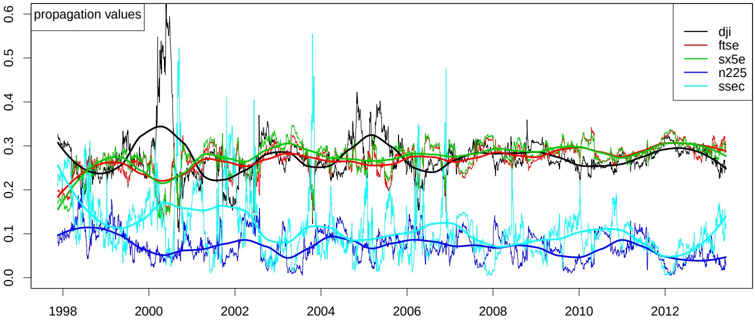
Time-varying characteristics

- Algorithm: Given a spillover matrix, find a VAR process with this spillover matrix.
- Connect processes with different properties together.
- Tool to study joint spillover / speed of convergence behavior.
- Does an increase in spillover imply an increase in speed of convergence? — No!

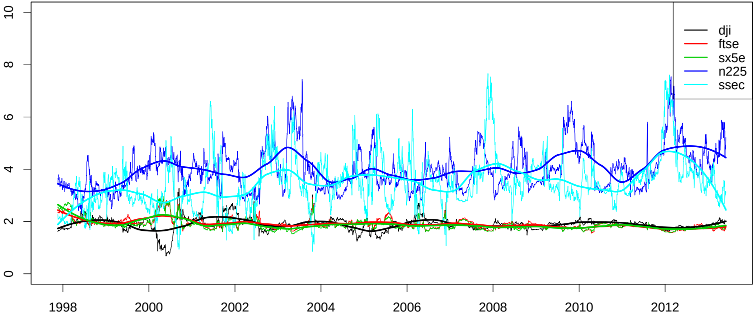
Time-varying characteristics



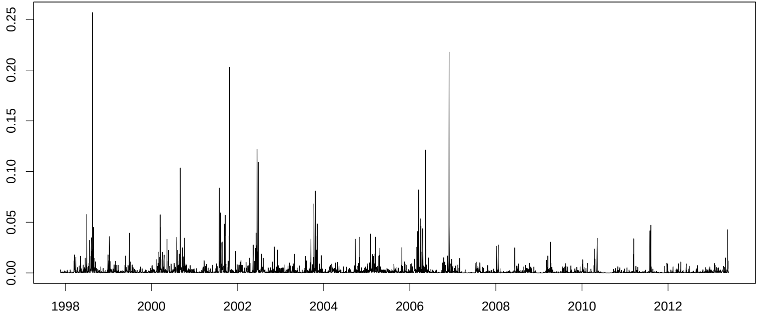
Progagation values



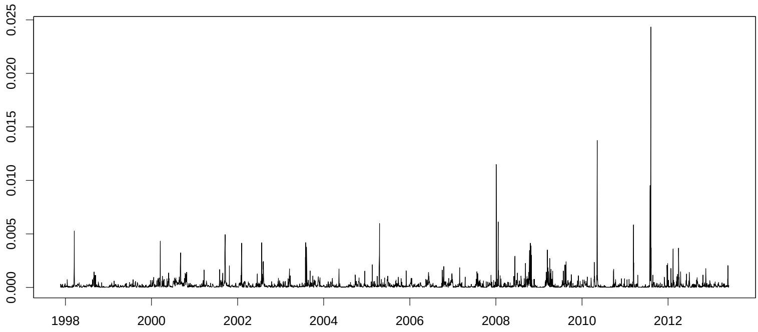
Relative entropy, different shock origins (hypothetical)



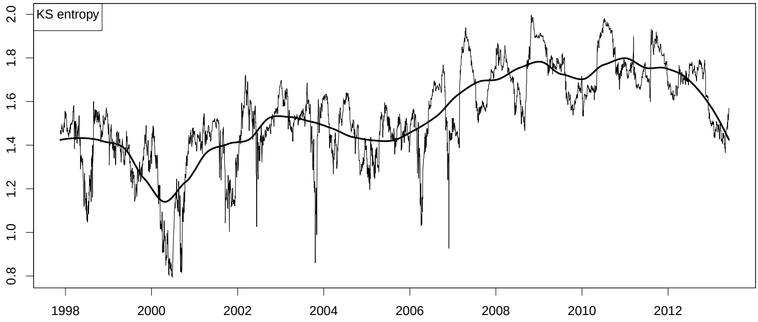
Relative entropy, stationary distributions (actual)



Relative entropy, non-stationary distributions (actual)



Kolmogorov-Sinai entropy



- The spillover approach lends itself to further network-related techniques.
- New tools can reveal further details in information flow.
- An increase in spillover index does not necessarily imply an increase in network stability.