Market Connectedness: Spillovers and More

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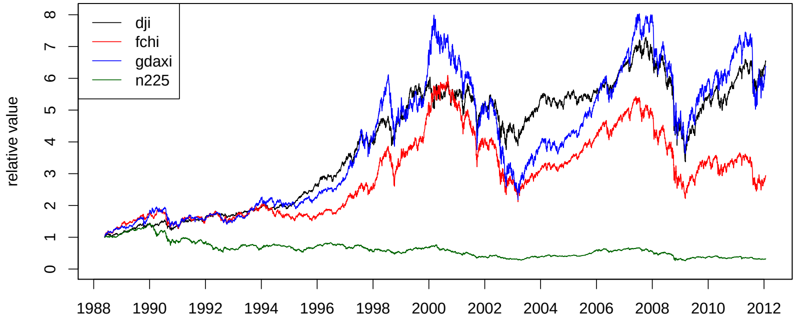
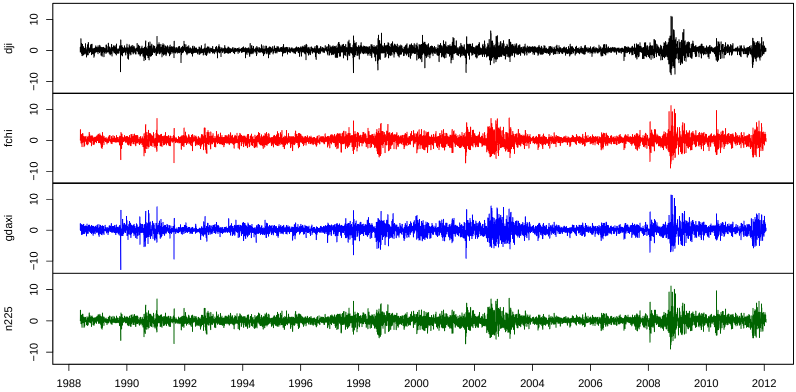
Market connectedness and fevd

- Assessing the degree of connectedness of equity markets
- Diebold & Yilmaz, 2005–2012:
  - VAR model for return series
  - forecast error variance decomposition (fevd)
  - spillover table
  - collapsed into the spillover index
- Our contribution: broadening the scope.
  - size of shocks propagated across markets?
  - location of shocks?

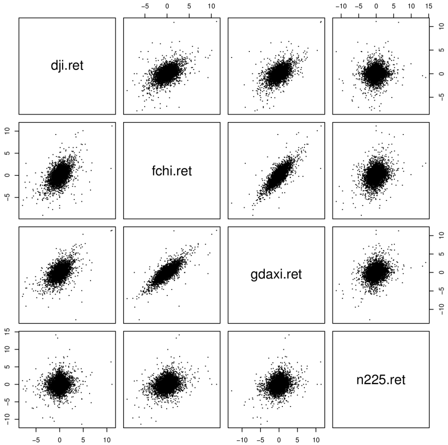
The level series (1988-05-23 ≡ 1)



The return series



Scatterplots of returns



Structural (= "primitive") form

Here: lag 1, omitting the constants

$$
\begin{aligned}
x_{1t} &= -b_{12}x_{2t} + \gamma_{11}x_{1t-1} + \gamma_{12}x_{2t-1} + \epsilon_{1t} \\
x_{2t} &= -b_{21}x_{1t} + \gamma_{21}x_{1t-1} + \gamma_{22}x_{2t-1} + \epsilon_{2t}
\end{aligned}
$$

Equivalently:

$$
\mathrm{B} \cdot x_t = \Gamma_1 \cdot x_{t-1} + \epsilon_t
$$

with

$$
x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix}, \quad \mathrm{B} = \begin{pmatrix} 1 & b_{12} \\ b_{21} & 1 \end{pmatrix}, \quad \Gamma_1 = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix},
$$

(ϵ_t): bivariate white noise, uncorrelated

Standard form

Re-writing the model:

$$x_t = \mathtt{A}_1 x_{t-1} + e_t$$

with

$$\mathtt{A}_1 = \mathtt{B}^{-1} \cdot \Gamma_1, \quad e_t = \mathtt{B}^{-1} \cdot \epsilon_t$$

- this is called the "standard form"
- seemingly, no more contemporaneous impact
- errors are correlated
- estimation, identification?

Impulse response functions: the case of AR(1)

- AR(1):

$$x_t = c + a\,x_{t-1} + \epsilon_t$$

- AR(1) as MA(∞):

$$x_t = \mu + \sum_{i=0}^{\infty} a^i\,\epsilon_{t-i} = \mu + \sum_{i=0}^{\infty} \Phi(i)\,\epsilon_{t-i}$$

- What is the effect of a one-unit shock in ϵ_t on x_t, x_{t+1}, x_{t+2}, $\ldots$?

Impulse response functions: the case of a VAR(1)

- Standard form:

$$x_t = \mathtt{A}_0 + \mathtt{A}_1\, x_{t-1} + e_t$$

- In MA(∞) representation:

$$x_t = \mu + \sum_{i=0}^{\infty} \mathtt{A}_1^i\, e_{t-i} = \mu + \sum_{i=0}^{\infty} \mathtt{A}_1^i\, \mathtt{B}^{-1}\, \epsilon_{t-i} = \mu + \sum_{i=0}^{\infty} \Phi(i)\, \epsilon_{t-i}$$

- Here, $\Phi(i)$ is a matrix:

$$\Phi(i) = \mathtt{A}_1^i \cdot \mathtt{B}^{-1} = \begin{pmatrix} \Phi_{11}(i) & \Phi_{12}(i) \\ \Phi_{21}(i) & \Phi_{22}(i) \end{pmatrix}$$

- Each $i \mapsto \Phi_{jk}(i)$ is called an impulse response function (irf).
- Depends on model identification.
- Effect of a one-unit shock in ϵ_t on $x_t, x_{t+1}, x_{t+2}, \ldots$?

The problem of identification

- The structural form is needed for the irfs.
- The structural form cannot be estimated directly.
- One solution: impose "causal priorities" on the structural model.
  (Sims, 1980, "orthogonalized" irf)
- Problem: irfs depend on priorities.
- Pesaran and Shin approach:

  > *To identify the irf of a component,*
  > *give that component highest priority.*

  (Pesaran & Shin, 1997)

Estimation results. . .

. . . for 1989-10-16 (using 50 days); output from R package vars

```
Estimation results for equation dji:
====================================
dji = dji.l1 + fchi.l1 + gdaxi.l1 + n225.l1

         Estimate Std. Error t value Pr(>|t|)
dji.l1    -0.1310     0.1584  -0.827   0.4129
fchi.l1    0.1558     0.2729   0.571   0.5709
gdaxi.l1  -0.2765     0.2296  -1.204   0.2348
n225.l1    0.7496     0.3752   1.998   0.0518 .
---

Residual standard error: 1.217 on 45 degrees of freedom
Multiple R-Squared: 0.1624,Adjusted R-squared: 0.0879
F-statistic:  2.18 on 4 and 45 DF,  p-value: 0.08642
```

```
Estimation results for equation gdaxi:
======================================
gdaxi = dji.l1 + fchi.l1 + gdaxi.l1 + n225.l1

         Estimate Std. Error t value Pr(>|t|)
dji.l1     1.4008     0.1331  10.521 1.04e-13 ***
fchi.l1   -0.5258     0.2293  -2.293   0.0266 *
gdaxi.l1   0.3355     0.1929   1.739   0.0889 .
n225.l1   -0.3622     0.3153  -1.149   0.2567
---

Residual standard error: 1.023 on 45 degrees of freedom
Multiple R-Squared: 0.7618,Adjusted R-squared: 0.7406
F-statistic: 35.97 on 4 and 45 DF,  p-value: 1.743e-13
```

```
Estimation results for equation fchi:
=====================================
fchi = dji.l1 + fchi.l1 + gdaxi.l1 + n225.l1

         Estimate Std. Error t value Pr(>|t|)
dji.l1    0.73736    0.09554   7.717 8.91e-10 ***
fchi.l1  -0.18646    0.16456  -1.133    0.263
gdaxi.l1  0.07331    0.13844   0.530    0.599
n225.l1  -0.07784    0.22625  -0.344    0.732
---

Residual standard error: 0.7338 on 45 degrees of freedom
Multiple R-Squared: 0.6104,Adjusted R-squared: 0.5758
F-statistic: 17.63 on 4 and 45 DF,  p-value: 9.065e-09
```

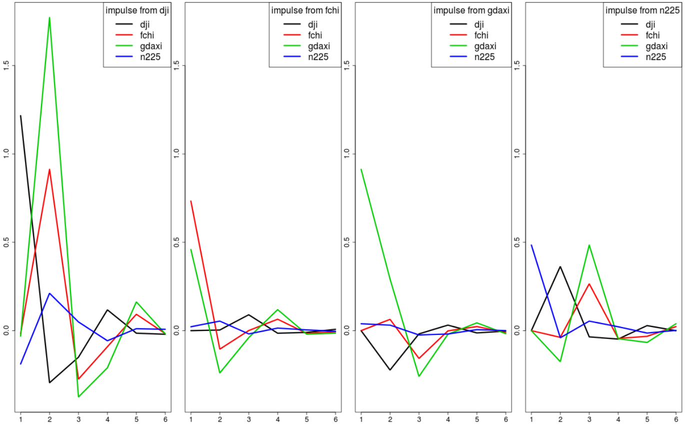
```
Estimation results for equation n225:
=====================================
n225 = dji.l1 + fchi.l1 + gdaxi.l1 + n225.l1

         Estimate Std. Error t value Pr(>|t|)
dji.l1    0.16269    0.06799   2.393    0.021 *
fchi.l1   0.05246    0.11710   0.448    0.656
gdaxi.l1  0.03776    0.09851   0.383    0.703
n225.l1  -0.08326    0.16100  -0.517    0.608
---

Residual standard error: 0.5222 on 45 degrees of freedom
Multiple R-Squared: 0.171,Adjusted R-squared: 0.09728
F-statistic:  2.32 on 4 and 45 DF,  p-value: 0.07133
```

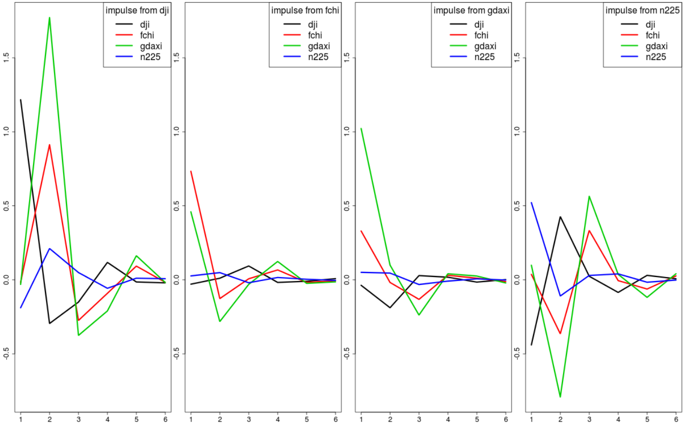
Impulse response functions. . .

. . . for 1989-10-16, when imposing "causal priorities"



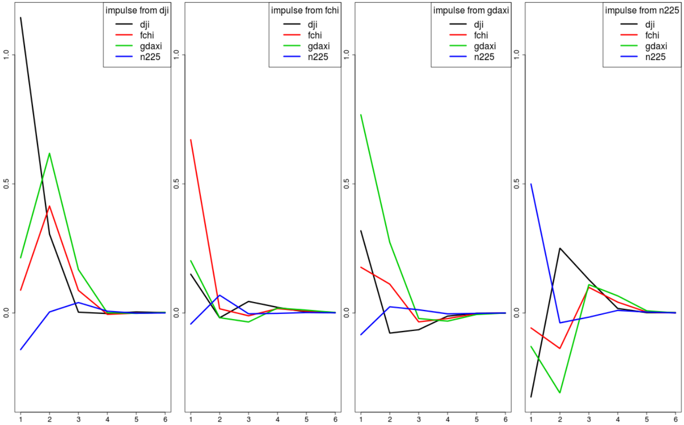
Impulse response functions. . .

. . . for 1989-10-16, Pesaran-Shin approach



Impulse response functions. . .

. . . for 1989-10-13, Pesaran-Shin approach



Forecasts and forecast errors

- Given $\mathtt{A}_0$, $\mathtt{A}_1$, x_t, forecast x_{t+i}!

$i = 1$:

forecast	$\hat{x}_{t+1} = \mathtt{A}_0 + \mathtt{A}_1 x_t$
forecast error	e_{t+1}

$i = 2$:

forecast	$\hat{x}_{t+2} = (1 + \mathtt{A}_1)\ \mathtt{A}_0 + \mathtt{A}_1^2 x_t$
forecast error	$e_{t+2} + \mathtt{A}_1 e_{t+1}$

$i = n$:

forecast	$\hat{x}_{t+n} = \left(1 + \mathtt{A}_1 + \cdots + \mathtt{A}_1^{n+1}\right)\ \mathtt{A}_0 + \mathtt{A}_1^n x_t$
forecast error	$e_{t+n} + \mathtt{A}_1 e_{t+n-1} + \mathtt{A}_1^2 e_{t+n-2} + \cdots + \mathtt{A}_1^{n-1} e_{t+1}$

Forecasts and forecast errors

- The forecast error in terms of the structural model:

$$\sum_{i=0}^{n-1} \mathtt{A}_1^i \, e_{t+n-i} = \sum_{i=0}^{n-1} \mathtt{A}_1^i \, \mathtt{B}^{-1} \, \epsilon_{t+n-i} = \sum_{i=0}^{n-1} \Phi(i) \, \epsilon_{t+n-i}$$

- For $x_{1\,t+n}$, the forecast error will be:

$$\sum_{i=0}^{n-1} \left(\Phi_{11}(i) \quad \Phi_{12}(i) \right) \begin{pmatrix} \epsilon_{1\,t+n-i} \\ \epsilon_{2\,t+n-i} \end{pmatrix}$$

$$\begin{array}{rcccccccc} = & \Phi_{11}(0) & \epsilon_{1\,t+n} & + & \Phi_{12}(0) & \epsilon_{2\,t+n} & + \\ & \vdots & & & \vdots & & \\ & \Phi_{11}(n-1) & \epsilon_{1\,t+1} & + & \Phi_{12}(n-1) & \epsilon_{2\,t+1} & \end{array}$$

Decomposition of the forecast error variance

For $x_{1\,t+n}$, the forecast error variance is:

$$\begin{array}{rcccccc}
\text{var}\left(\sum_{i=0}^{n-1} \left(\Phi_{11}(i) \;\; \Phi_{12}(i)\right) \begin{pmatrix} \epsilon_{1\,t+n-i} \\ \epsilon_{2\,t+n-i} \end{pmatrix}\right) & & & & & & \\
= & \Phi_{11}^2(0) & \sigma_1^2 & + & \Phi_{12}^2(0) & \sigma_2^2 & + \\
 & \vdots & & & \vdots & & \\
 & \Phi_{11}^2(n-1) & \sigma_1^2 & + & \Phi_{12}^2(n-1) & \sigma_2^2 & \\
= & \multicolumn{2}{c}{\underbrace{\sigma_1^2 \cdot \sum_{i=0}^{n-1} \Phi_{11}^2(i)}} & + & \multicolumn{2}{c}{\underbrace{\sigma_2^2 \cdot \sum_{i=0}^{n-1} \Phi_{12}^2(i)}} & \\
\text{due to}\ldots & \multicolumn{2}{c}{\text{shocks in } \epsilon_{1t}} & & \multicolumn{2}{c}{\text{shocks in } \epsilon_{2t}} &
\end{array}$$

Forecast error variance decomposition — the case of dji

- Forecast error variance (with Φ = an irf):

$$\text{var}\left\{ \sum_{i=0}^{n-1} \left(\Phi_{\text{dji}}^{\text{dji}}(i),\ \Phi_{\text{dji}}^{\text{fchi}}(i),\ \Phi_{\text{dji}}^{\text{gdaxi}}(i),\ \Phi_{\text{dji}}^{\text{n225}}(i) \right) \times \begin{pmatrix} \epsilon_{\text{dji}\ \ ,t+n-i} \\ \epsilon_{\text{fchi}\ \ ,t+n-i} \\ \epsilon_{\text{gdaxi},t+n-i} \\ \epsilon_{\text{n225}\ ,t+n-i} \end{pmatrix} \right\}$$

- This expression equals

$$\sigma_{\text{dji}}^2 \cdot \textstyle\sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{dji}})^2(i) \quad + \quad \sigma_{\text{fchi}}^2 \cdot \textstyle\sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{fchi}})^2(i) \quad +$$

$$\sigma_{\text{gdaxi}}^2 \cdot \textstyle\sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{gdaxi}})^2(i) \quad + \quad \sigma_{\text{n225}}^2 \cdot \textstyle\sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{n225}})^2(i)$$

Spillover table & spillover index

- Spillover table, schematically:

		from (time t)			
		dji	fchi	gdaxi	n225
to (time $t+n$)	dji	$\square$	$\blacksquare$	$\blacksquare$	$\blacksquare$
	fchi	$\blacksquare$	$\square$	$\blacksquare$	$\blacksquare$
	gdaxi	$\blacksquare$	$\blacksquare$	$\square$	$\blacksquare$
	n225	$\blacksquare$	$\blacksquare$	$\blacksquare$	$\square$

- Spillover index $= \dfrac{\sum \blacksquare}{\sum \blacksquare + \sum \square}$ (Diebold & Yilmaz, 2005)

Example: 1989-10-16

date	to:	spillover matrix from... dji	fchi	gdaxi	n225	index
1989-10-13	dji	0.817	0.015	0.065	0.108	29.271%
	fchi	0.261	0.629	0.063	0.047	
	gdaxi	0.352	0.033	0.514	0.100	
	n225	0.076	0.023	0.027	0.874	
1989-10-16	dji	0.787	0.005	0.019	0.189	49.205%
	fchi	0.497	0.300	0.069	0.134	
	gdaxi	0.583	0.054	0.194	0.169	
	n225	0.225	0.010	0.015	0.750	
1989-10-17	dji	0.783	0.004	0.006	0.207	49.062%
	fchi	0.468	0.370	0.089	0.074	
	gdaxi	0.584	0.049	0.176	0.191	
	n225	0.276	0.002	0.014	0.709	

Example: 2012-08-20

date	to:	spillover matrix from... dji	fchi	gdaxi	n225	index
2012-08-17	dji	0.416	0.282	0.272	0.029	61.945%
	fchi	0.256	0.362	0.324	0.057	
	gdaxi	0.253	0.333	0.365	0.049	
	n225	0.185	0.252	0.184	0.379	
2012-08-20	dji	0.412	0.286	0.272	0.029	62.151%
	fchi	0.261	0.360	0.323	0.057	
	gdaxi	0.256	0.332	0.363	0.049	
	n225	0.187	0.250	0.185	0.379	
2012-08-21	dji	0.421	0.281	0.267	0.031	61.718%
	fchi	0.257	0.360	0.324	0.059	
	gdaxi	0.252	0.333	0.365	0.050	
	n225	0.180	0.250	0.185	0.386	

Example: 1991-07-03

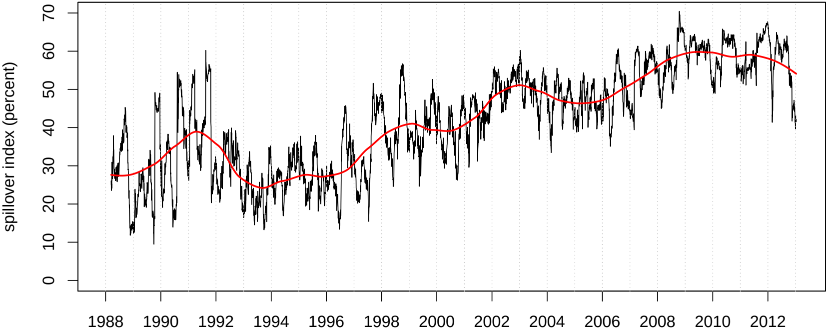
date	to:	spillover matrix from... dji	fchi	gdaxi	n225	index
1991-07-02	dji	0.617	0.116	0.100	0.167	38.448%
	fchi	0.116	0.585	0.193	0.106	
	gdaxi	0.069	0.221	0.655	0.055	
	n225	0.164	0.119	0.113	0.604	
1991-07-03	dji	0.597	0.133	0.088	0.183	39.993%
	fchi	0.128	0.561	0.172	0.139	
	gdaxi	0.072	0.217	0.652	0.059	
	n225	0.178	0.147	0.085	0.590	
1991-07-04	dji	0.608	0.130	0.086	0.177	39.681%
	fchi	0.125	0.561	0.174	0.140	
	gdaxi	0.072	0.215	0.654	0.058	
	n225	0.171	0.149	0.090	0.590	

Example: 2004-03-02

date	to:	spillover matrix from... dji	fchi	gdaxi	n225	index
2004-03-01	dji	0.687	0.104	0.176	0.033	40.753%
	fchi	0.085	0.484	0.358	0.073	
	gdaxi	0.135	0.351	0.455	0.058	
	n225	0.036	0.130	0.090	0.744	
2004-03-02	dji	0.721	0.089	0.151	0.040	40.002%
	fchi	0.082	0.484	0.368	0.066	
	gdaxi	0.121	0.364	0.457	0.057	
	n225	0.037	0.130	0.095	0.738	
2004-03-03	dji	0.701	0.101	0.155	0.043	40.812%
	fchi	0.094	0.476	0.365	0.066	
	gdaxi	0.129	0.362	0.452	0.057	
	n225	0.041	0.127	0.094	0.738	

Example: dji, fchi, gdaxi, n225

Spillover index series:



Limitations

- Spillover index: summary measure.
- Four hypothetical spillover matrices:

$$\begin{array}{cccc} \text{(a)} & \text{(b)} & \text{(c)} & \text{(d)} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0.6 \\ 0 & 0.6 & 0.4 \end{pmatrix} & \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{pmatrix} & \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{pmatrix} & \begin{pmatrix} 0.7 & 0 & 0.3 \\ 0.4 & 0.6 & 0 \\ 0.3 & 0.2 & 0.5 \end{pmatrix} \end{array}$$

- spillover index: 40%

- spillovers: very different!

Our interpretation of the spillover matrix

- The spillover matrix is the most recent information for a day.
- We consider the spread of a hypothetical shock ("news", "information") through the network defined by the spillover matrix.
- A shock to node (or market) i on day t is given by

$$\mathbf{n}_0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{array}{l} \\ \\ \\ \leftarrow i\text{-th component} \\ \\ \\ \\ \end{array}$$

- What happens when such a shock hits the network?

Shocks

- What happens when such a shock hits the network?
- Two phenomena can be studied:
  - the relative size of shocks, as they travel through the network
  - the location of a shock, and the time share it spends in a given node

Shock sizes

- Assumptions
  - Given: A spillover matrix $\mathbf{M}_t$ for day t
  - Propagation of a shock within day $t+1$:
    - initial shock size: $\mathbf{n}_0$ (a unit vector)
    - shock propagation in short time interval according to

$$\mathbf{n}_{s+1} = \mathbf{M}_t \cdot \mathbf{n}_s, \quad s = 0, 1, \ldots$$

- Question: What happens to shock size $\mathbf{n}_s$ as $s \to \infty$?

Example: propagation values

step number	$s=1$			$s=2$			$s=\infty$		
origin of shock	1	2	3	1	2	3	1	2	3
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0.6 \\ 0 & 0.6 & 0.4 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.4 \\ 0.6 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.6 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.52 \\ 0.48 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.48 \\ 0.52 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$
$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.6 \\ 0.1 \\ 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.6 \\ 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.3 \\ 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.40 \\ 0.15 \\ 0.15 \end{bmatrix}$	$\begin{bmatrix} 0.30 \\ 0.47 \\ 0.38 \end{bmatrix}$	$\begin{bmatrix} 0.30 \\ 0.38 \\ 0.47 \end{bmatrix}$	$\begin{bmatrix} 0.20 \\ 0.20 \\ 0.20 \end{bmatrix}$	$\begin{bmatrix} 0.40 \\ 0.40 \\ 0.40 \end{bmatrix}$	$\begin{bmatrix} 0.40 \\ 0.40 \\ 0.40 \end{bmatrix}$
$\begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.5 \\ 0.9 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.5 \\ 0 \\ 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.10 \\ 0.09 \\ 0.09 \end{bmatrix}$	$\begin{bmatrix} 0.45 \\ 0.86 \\ 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.45 \\ 0.05 \\ 0.86 \end{bmatrix}$	$\begin{bmatrix} 0.09 \\ 0.09 \\ 0.09 \end{bmatrix}$	$\begin{bmatrix} 0.45 \\ 0.45 \\ 0.45 \end{bmatrix}$	$\begin{bmatrix} 0.45 \\ 0.45 \\ 0.45 \end{bmatrix}$
$\begin{bmatrix} 0.7 & 0 & 0.3 \\ 0.4 & 0.6 & 0 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.7 \\ 0.4 \\ 0.3 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.6 \\ 0.2 \end{bmatrix}$	$\begin{bmatrix} 0.3 \\ 0 \\ 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.58 \\ 0.52 \\ 0.44 \end{bmatrix}$	$\begin{bmatrix} 0.06 \\ 0.36 \\ 0.22 \end{bmatrix}$	$\begin{bmatrix} 0.36 \\ 0.12 \\ 0.34 \end{bmatrix}$	$\begin{bmatrix} 0.53 \\ 0.53 \\ 0.53 \end{bmatrix}$	$\begin{bmatrix} 0.16 \\ 0.16 \\ 0.16 \end{bmatrix}$	$\begin{bmatrix} 0.32 \\ 0.32 \\ 0.32 \end{bmatrix}$

Example: propagation values

- It holds that:

  The relative size of a shock, as $s \to \infty$, is determined by the left eigenvector of the spillover matrix.

- left eigenvector: value of a shock to which the market is exposed as seed for future variability or risk ("propagation value")

Example: 1989-10-16

date	to:	spillover matrix from... dji	fchi	gdaxi	n225	index	propagation values of... dji	fchi	gdaxi	n225
1989-10-13	dji	0.817	0.015	0.065	0.108	29.271%	0.417	0.052	0.087	0.444
	fchi	0.261	0.629	0.063	0.047					
	gdaxi	0.352	0.033	0.514	0.100					
	n225	0.076	0.023	0.027	0.874					
1989-10-16	dji	0.787	0.005	0.019	0.189	49.205%	0.539	0.011	0.022	0.428
	fchi	0.497	0.300	0.069	0.134					
	gdaxi	0.583	0.054	0.194	0.169					
	n225	0.225	0.010	0.015	0.750					
1989-10-17	dji	0.783	0.004	0.006	0.207	49.062%	0.569	0.006	0.012	0.413
	fchi	0.468	0.370	0.089	0.074					
	gdaxi	0.584	0.049	0.176	0.191					
	n225	0.276	0.002	0.014	0.709					

Example: 2012-08-20

date	to:	spillover matrix from... dji	fchi	gdaxi	n225	index	propagation values of... dji	fchi	gdaxi	n225
2012-08-17	dji	0.416	0.282	0.272	0.029	61.945%	0.298	0.322	0.312	0.068
	fchi	0.256	0.362	0.324	0.057					
	gdaxi	0.253	0.333	0.365	0.049					
	n225	0.185	0.252	0.184	0.379					
2012-08-20	dji	0.412	0.286	0.272	0.029	62.151%	0.300	0.322	0.311	0.068
	fchi	0.261	0.360	0.323	0.057					
	gdaxi	0.256	0.332	0.363	0.049					
	n225	0.187	0.250	0.185	0.379					
2012-08-21	dji	0.421	0.281	0.267	0.031	61.718%	0.299	0.320	0.310	0.071
	fchi	0.257	0.360	0.324	0.059					
	gdaxi	0.252	0.333	0.365	0.050					
	n225	0.180	0.250	0.185	0.386					

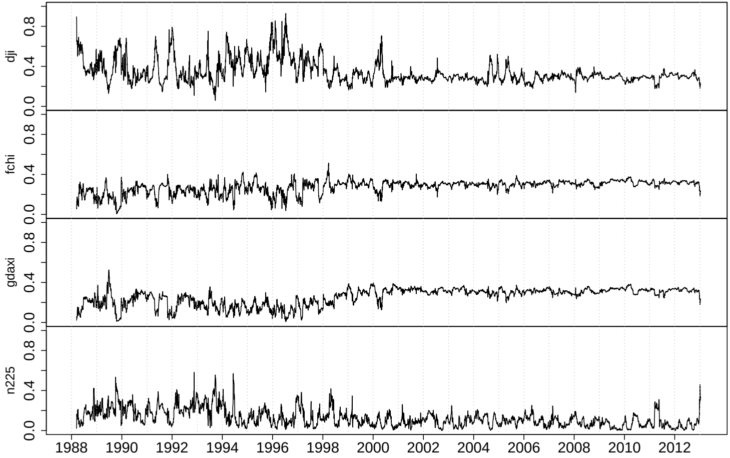
Example: 1991-07-03

date	to:	spillover matrix from... dji	fchi	gdaxi	n225	index	propagation values of... dji	fchi	gdaxi	n225
1991-07-02	dji	0.617	0.116	0.100	0.167	38.448%	0.225	0.277	0.289	0.209
	fchi	0.116	0.585	0.193	0.106					
	gdaxi	0.069	0.221	0.655	0.055					
	n225	0.164	0.119	0.113	0.604					
1991-07-03	dji	0.597	0.133	0.088	0.183	39.993%	0.236	0.275	0.253	0.235
	fchi	0.128	0.561	0.172	0.139					
	gdaxi	0.072	0.217	0.652	0.059					
	n225	0.178	0.147	0.085	0.590					
1991-07-04	dji	0.608	0.130	0.086	0.177	39.681%	0.236	0.275	0.257	0.232
	fchi	0.125	0.561	0.174	0.140					
	gdaxi	0.072	0.215	0.654	0.058					
	n225	0.171	0.149	0.090	0.590					

Example: 2004-03-02

date	to:	spillover matrix from... dji	fchi	gdaxi	n225	index	propagation values of... dji	fchi	gdaxi	n225
2004-03-01	dji	0.687	0.104	0.176	0.033	40.753%	0.229	0.293	0.297	0.181
	fchi	0.085	0.484	0.358	0.073					
	gdaxi	0.135	0.351	0.455	0.058					
	n225	0.036	0.130	0.090	0.744					
2004-03-02	dji	0.721	0.089	0.151	0.040	40.002%	0.237	0.293	0.295	0.174
	fchi	0.082	0.484	0.368	0.066					
	gdaxi	0.121	0.364	0.457	0.057					
	n225	0.037	0.130	0.095	0.738					
2004-03-03	dji	0.701	0.101	0.155	0.043	40.812%	0.241	0.291	0.292	0.176
	fchi	0.094	0.476	0.365	0.066					
	gdaxi	0.129	0.362	0.452	0.057					
	n225	0.041	0.127	0.094	0.738					

Example: dji, fchi, gdaxi, n225



A shock traveling through the network

- A shock hits a node (market, asset) of the network.
- This shock (always unit size) travels through the network.
- Shock dynamics: determined by spillover table (similar to Markov chain)
- Phenomena to study:
  - "information equilibrium" or "news balance" (stationary distribution of shock location)
  - disturbance of news balance, when a shock hits
  - speed of convergence to news balance, when a shock hits
  - information gain from day to day

The spillover table

- Can we use the spillover table as a Markov transition matrix?
- $\mathbf{M}_t$ is row-stochastic: If p (column vector) is a probability distribution, then:
  - $\mathbf{M}_t \cdot p$ need not be a distribution
  - $p' \cdot \mathbf{M}_t$ is a distribution
- However, a Markov chain with $p'_{s+1} = p'_s \cdot \mathbf{M}_t$ is running backward in time (relative to the setup of $\mathbf{M}_t$).
- If p is the distribution of shock location, the $p' \cdot \mathbf{M}_t$ is the distribution of shock origin (not: shock evolution)
- Time needs to be reversed.

Transformation of the spillover table

- Can we define a Markov chain running forward in time?
- For strongly connected networks only!
- Then, $\mathbf{M}_t$ can be transformed into a forward Markov transition matrix (using the eigenvalue structure):

$$\mathbf{V}_t^{-1} \cdot \mathbf{M}_t' \cdot \mathbf{V}_t$$

(Tuljapurkar, 1982)

- A Markov chain with $p_{s+1}' = p_s' \cdot \mathbf{V}_t^{-1} \cdot \mathbf{M}_t' \cdot \mathbf{V}_t$ is running forward in time.

Example: backward and forward transition matrix

spillover table	Markov transition matrix backward	Markov transition matrix forward
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0.6 \\ 0 & 0.6 & 0.4 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0.6 \\ 0 & 0.6 & 0.4 \end{bmatrix}$	[n.a.]
$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$
$\begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{bmatrix}$
$\begin{bmatrix} 0.7 & 0 & 0.3 \\ 0.4 & 0.6 & 0 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.7 & 0 & 0.3 \\ 0.4 & 0.6 & 0 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.7 & 0.12 & 0.18 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{bmatrix}$

- In general, backward and forward transition matrices are different.
- Exceptions: networks of N nodes, where $N-1$ nodes are indistinguishable with respect to spillovers

Given is one matrix $\mathbf{M}_t$ only.

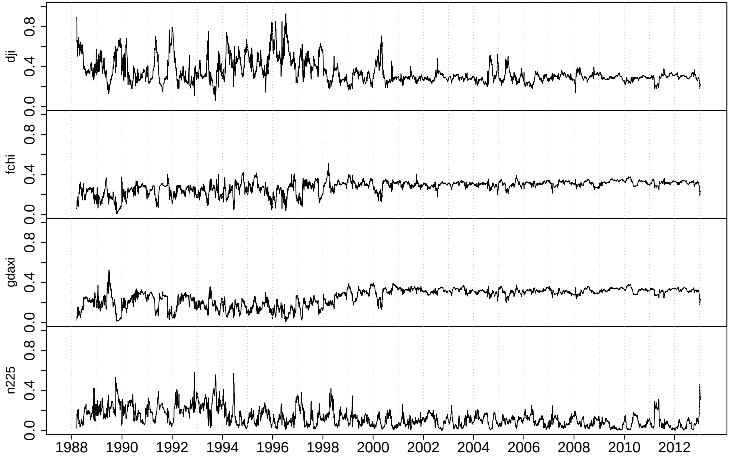
- If a shock hits a node (market, asset) of the network on day t:
  - Where will the shock be settling within day $t+1$?
  - What is the share of time the shock will spend in one of the network's nodes?
  - What is the stationary distribution of the Markov chain running forward in time?
- It can be shown that:

  The stationary probability distribution of the Markov chain running forward in time equals the (normed) left eigenvector of $\mathbf{M}_t$.
- Dual interpretation of propagation values!

Example: stationary probability distribution

step number	$s=1$			$s=2$			$s=\infty$		
origin of shock	1	2	3	1	2	3	1	2	3
$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.6 \\ 0.2 \\ 0.2 \end{bmatrix}$	$\begin{bmatrix} 0.1 \\ 0.6 \\ 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.1 \\ 0.3 \\ 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.15 \\ 0.47 \\ 0.38 \end{bmatrix}$	$\begin{bmatrix} 0.15 \\ 0.38 \\ 0.47 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$
$\begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.1 \\ 0.9 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.1 \\ 0 \\ 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.10 \\ 0.45 \\ 0.45 \end{bmatrix}$	$\begin{bmatrix} 0.09 \\ 0.86 \\ 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.09 \\ 0.05 \\ 0.86 \end{bmatrix}$	$\begin{bmatrix} 0.09 \\ 0.45 \\ 0.45 \end{bmatrix}$	$\begin{bmatrix} 0.09 \\ 0.45 \\ 0.45 \end{bmatrix}$	$\begin{bmatrix} 0.09 \\ 0.45 \\ 0.45 \end{bmatrix}$
$\begin{bmatrix} 0.7 & 0.12 & 0.18 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.7 \\ 0.12 \\ 0.18 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.6 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.58 \\ 0.16 \\ 0.26 \end{bmatrix}$	$\begin{bmatrix} 0.20 \\ 0.36 \\ 0.44 \end{bmatrix}$	$\begin{bmatrix} 0.60 \\ 0.06 \\ 0.34 \end{bmatrix}$	$\begin{bmatrix} 0.53 \\ 0.16 \\ 0.32 \end{bmatrix}$	$\begin{bmatrix} 0.53 \\ 0.16 \\ 0.32 \end{bmatrix}$	$\begin{bmatrix} 0.53 \\ 0.16 \\ 0.32 \end{bmatrix}$

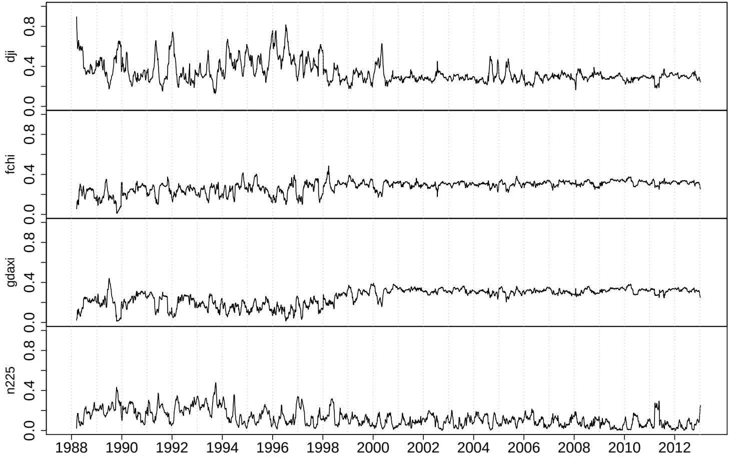
Example: dji, fchi, gdaxi, n225



Given is a sequence $(\mathbf{M}_t)_t$.

- Where is the day-0 news transmitted to? On day 1, 2, ...?
- The sequence $(\mathbf{V}_t^{-1} \cdot \mathbf{M}_t' \cdot \mathbf{V}_t)_t$ of forward transition matrices describes day-to-day news transmission.
- This defines an inhomogeneous Markov chain running forward in time steps of one day.
- A stationary probability distribution need not exist.
- How does the non-stationary probability distribution evolve day by day?

Example: dji, fchi, gdaxi, n225



Motivation

- Several probability distributions are associated with each day:
  - initial shock distribution (a unit vector)
  - stationary distribution of shock position
  - non-stationary distribution of shock position
- informational content of a distribution?
- distance between distributions?
  (for example, between initial shock distribution and stationary distribution)

Entropy & KLIC

- Given:
  - random variable X with distribution $\mathbb{P}$
  - random variable Y with distribution $\mathbb{Q}$
- Entropy of X:

$$H(X) = -\sum_{x} \mathbb{P}(x) \cdot \log_2 \mathbb{P}(x)$$

- Kullback-Leibler divergence (KLIC) of (false) $\mathbb{Q}$ from (true) $\mathbb{P}$:

$$D_{KL}(\mathbb{P} \parallel \mathbb{Q}) = \sum_{x} \mathbb{P}(x) \cdot \log_2 \frac{\mathbb{P}(x)}{\mathbb{Q}(x)}$$

Example: entropy

- Given: random variable X and its distribution $\mathbb{P}$

	A	B	C	D
$X, \mathbb{P}$	$1/2$	$1/4$	$1/8$	$1/8$

- Suppose you know the support $\{A, B, C, D\}$ and $\mathbb{P}$.
- Average number of ("clever") guesses required to identify a realization:

$$1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8} = 1.75$$

- This equals the entropy of X:

$$H(X) = -\frac{1}{2} \cdot \log_2 \frac{1}{2} - \frac{1}{4} \cdot \log_2 \frac{1}{4} - \frac{1}{8} \cdot \log_2 \frac{1}{8} - \frac{1}{8} \cdot \log_2 \frac{1}{8}$$

- Case of a uniform distribution: $H(X) = 2$.

Example: KLIC

- Given: random variables X, Y, their respective distributions $\mathbb{P}$, $\mathbb{Q}$

	A	B	C	D
X, $\mathbb{P}$	1/2	1/4	1/8	1/8
Y, $\mathbb{Q}$	1/8	1/8	1/4	1/2

- Suppose you know the support $\{A, B, C, D\}$ and $\mathbb{Q}$, but not $\mathbb{P}$.
- Average number of guesses required to identify a realization of X when using the coding for Y:

$$1 \cdot \frac{1}{8} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{4} + 3 \cdot \frac{1}{2} = 2.625 > 1.75$$

- The difference equals the Kullback-Leibler divergence of (false) $\mathbb{Q}$ from (true) $\mathbb{P}$:

$$D_{KL}(\mathbb{P} \parallel \mathbb{Q}) = \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 1 - \frac{1}{8} \cdot 1 - \frac{1}{8} \cdot 2 = 0.875$$

Example: KLIC

- Given: random variables X, Y, their respective distributions $\mathbb{P}$, $\mathbb{Q}$

	A	B	C	D
X, $\mathbb{P}$	1/2	1/4	1/8	1/8
Y, $\mathbb{Q}$	1/4	1/4	1/4	1/4

- Again, suppose you know the support $\{A, B, C, D\}$ and $\mathbb{Q}$, but not $\mathbb{P}$.
- Average number of guesses required to identify a realization of X when using the coding for Y:

$$2 > 1.75$$

- Again, the difference equals the Kullback-Leibler divergence of (false) $\mathbb{Q}$ from (true) $\mathbb{P}$:

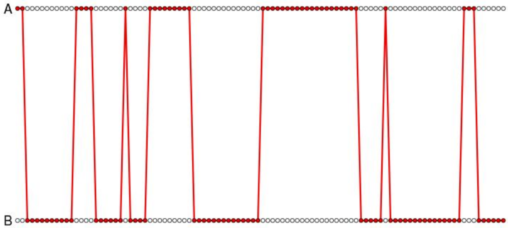
$$D_{KL}(\mathbb{P} \parallel \mathbb{Q}) = \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 0 - \frac{1}{8} \cdot 1 - \frac{1}{8} \cdot 1 = 0.25$$

Example: simulation of a Markov chain, I

Probabilities, entropy:

From:	To: A	B	steady-state	KLIC: initial-state ‖ steady-state
A	0.9	0.1	0.5	1.000
B	0.1	0.9	0.5	1.000

Simulation results:

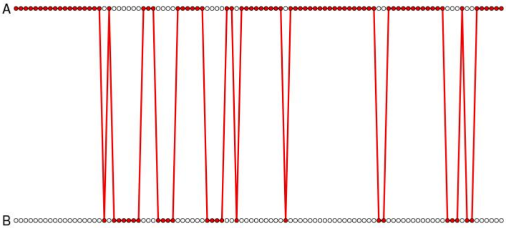


Example: simulation of a Markov chain, II

Probabilities, entropy:

From:	To: A	B	steady-state	KLIC: initial-state ‖ steady-state
A	0.9	0.1	0.75	0.415
B	0.3	0.7	0.25	2.000

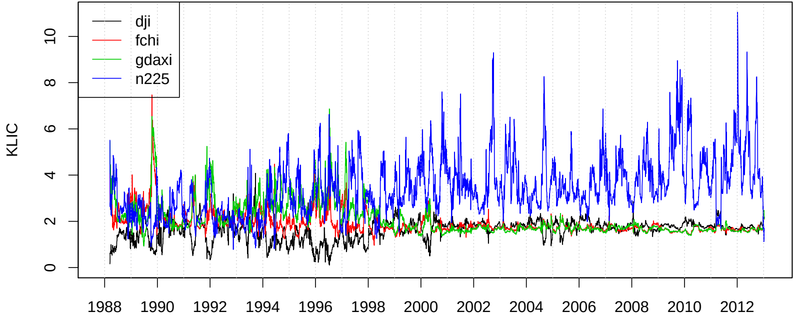
Simulation results:



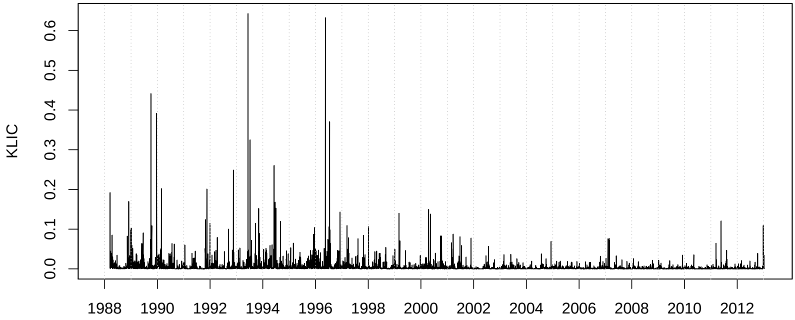
Example: KLIC of a shock w.r.t. the stationary distribution

Entropy measure origin of shock	KLIC 1	KLIC 2	KLIC 3
$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	2.322	1.322	1.322
$\begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{bmatrix}$	3.459	1.138	1.138
$\begin{bmatrix} 0.7 & 0.12 & 0.18 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{bmatrix}$	0.926	2.663	1.663

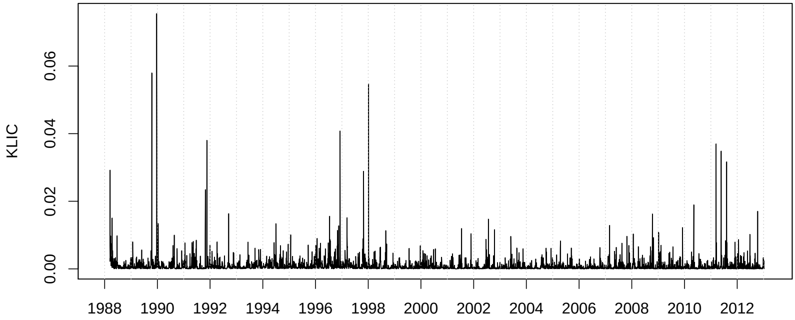
KLIC: today's stationary distribution and the case when a shock comes from...



KLIC today w.r.t. yesterday: stationary distributions



KLIC today w.r.t. yesterday: non-stationary distributions



Motivation

- Again a single-day perspective.
- If a shock hits a node (market, asset) of the network on day t: How fast will the shock be digested within day $t+1$?
- Speed of convergence of the Markov chain on that day?

Kolmogorov-Sinai entropies

- Given:
  - a Markov chain with forward transition matrix $\mathbf{P} = (p_{ij})$
  - its stationary probability distribution $\mathbb{P}$
- Kolmogorov-Sinai entropy, standard forward

$$\mathrm{KS} = -\sum_{i,j} \mathbb{P}(i) \cdot \log_2 \left(p_{ij}^{p_{ij}} \right)$$

- Kolmogorov-Sinai entropy, reversed time:

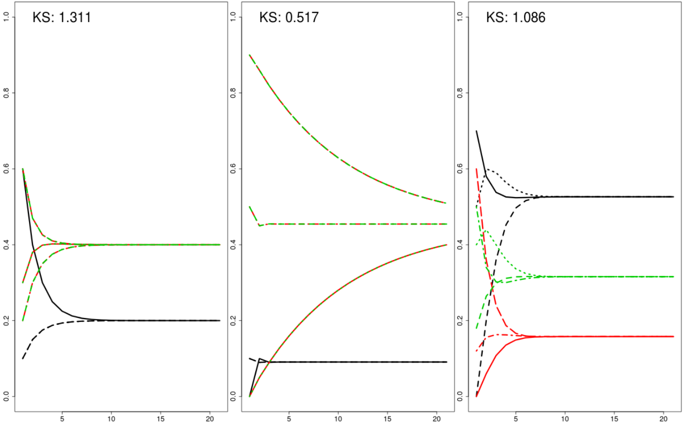
$$\mathrm{KSR} = -\sum_{i,j} \mathbb{P}(i) \cdot \log_2 \left(p_{ji}^{p_{ij}} \right)$$

- Entropy production: KSR − KS

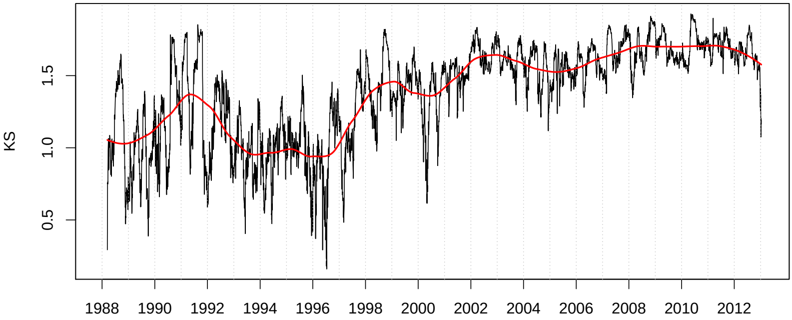
Example: Kolmogorov-Sinai entropies

Entropy measure	KS	KSR
$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	1.311	1.311
$\begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{bmatrix}$	0.517	0.517
$\begin{bmatrix} 0.7 & 0.12 & 0.18 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{bmatrix}$	1.086	[n.a.]

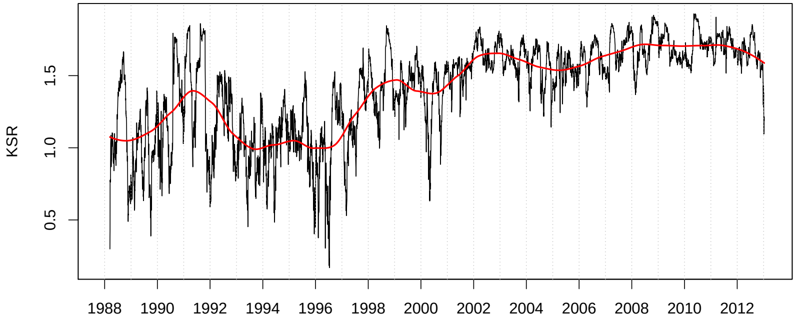
Example: Speed of convergence



Kolmogorov-Sinai entropy, standard forward



Kolmogorov-Sinai entropy, reversed time



Kolmogorov-Sinai entropy production