# EC 613:
# Advanced Topics in Financial Econometrics

# FEC 514:
# Applications in Financial Modeling

Harald Schmidbauer

İSTANBUL BİLGİ ÜNİVERSİTESİ

# About These Slides

- The present slides are not self-contained; they need to be explained and discussed.

- Even though being a “work in progress” and subject to revision, the slides constitute copyrighted material.
  If you want to reproduce or copy anything from the slides, please ask:



| | |
|---|---|
| Harald Schmidbauer | **harald** at **hs-stat** dot **com** |
| Angi Rösch | **angi** at **angi-stat** dot **com** |



- The slides were produced using LaTeX (www.latex-project.org) and R (the R project; www.R-project.org) on a GNU/Linux system.

- R files used for this course are available upon request.

# Chapter 2:

# Analyzing Price Changes: Some Aspects

# 2.1 Introduction

## The starting point.

- Typical starting point: sequence $(P_t)_t$ of price quotes
- Often more interesting: price changes, i.e. returns

$$\text{simple return:} \qquad R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad \text{or} \quad \frac{P_t - P_{t-1}}{P_{t-1}} \cdot 100\%$$

$$\text{log return:} \qquad r_t = \ln \frac{P_t}{P_{t-1}}$$

$$\text{gross return:} \qquad 1 + R_t = \frac{P_t}{P_{t-1}}$$

# 2.1 Introduction

## The starting point.

- The analysis of $(R_t)$ or $(r_t)$ or $(1 + R_t)$ can then:
  - either neglect time
    (good for understanding the average behaviour of the series)
  - or take time explicitly into account
    (good for understanding the conditional behaviour of the series)

- In any case, we need the methods of statistics!

- A very brief review of the objectives of statistical analysis is therefore in order.

# 2.2 Statistics: Reasoning With Numbers

## The goals of descriptive and inductive statistics.

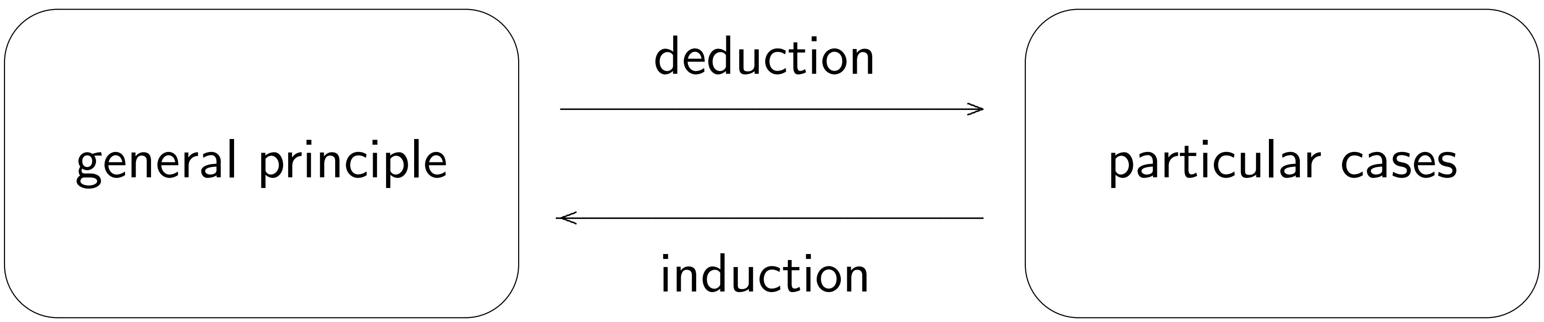
The goal of. . .

- . . . descriptive statistics is: Describe, summarize, display given data (data reduction!).
- . . . inductive statistics is: Draw conclusions from data (observations) to more general principles.
  The process of drawing conclusions is called statistical inference.

# 2.2 Statistics: Reasoning With Numbers

## Conclusions.

There are two kinds of conclusions:




- In the context of inductive statistics, the "particular cases" are observed data (sample data).
- The "general principle" is a probability distribution, characterizing the entire population.

# 2.2 Statistics: Reasoning With Numbers

## Inductive statistics.

- The paradigm of inductive statistics is:

**Regard the observations as the outcome of a random experiment, that is, as being produced by a stochastic model.**

- Stochastic model: a mathematical model on the basis of probability.
- The object of research is then the stochastic model, rather than the observations!

# 2.2 Statistics: Reasoning With Numbers

## Observations and stochastic models: analogies and their relation.

OBSERVATIONS

- variables
- observed values
- relative frequency
- empirical distribution

statistical inference →

← 

- random experiments
- computer simulations

STOCHASTIC MODEL

- random variables
- their realizations
- probability
- probability distribution

# 2.3 Data and Stochastic Models

## The role of stochastic modeling: two examples.

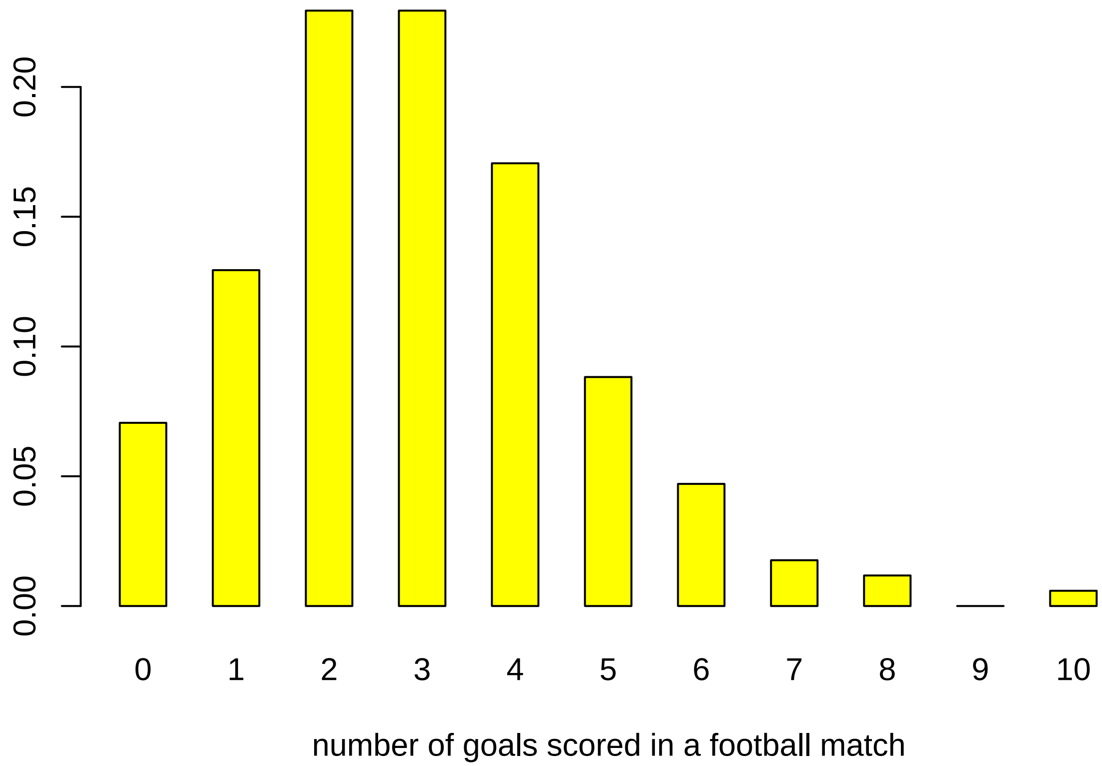
Now, for two simple examples, we shall:

- see the empirical evidence in the form of a chart,
- add a plot of a suitable stochastic model to the chart,
- discuss briefly what the stochastic model can tell us.

# 2.3 Data and Stochastic Models

Example 1: The number of goals scored in matches of Beşiktaş.

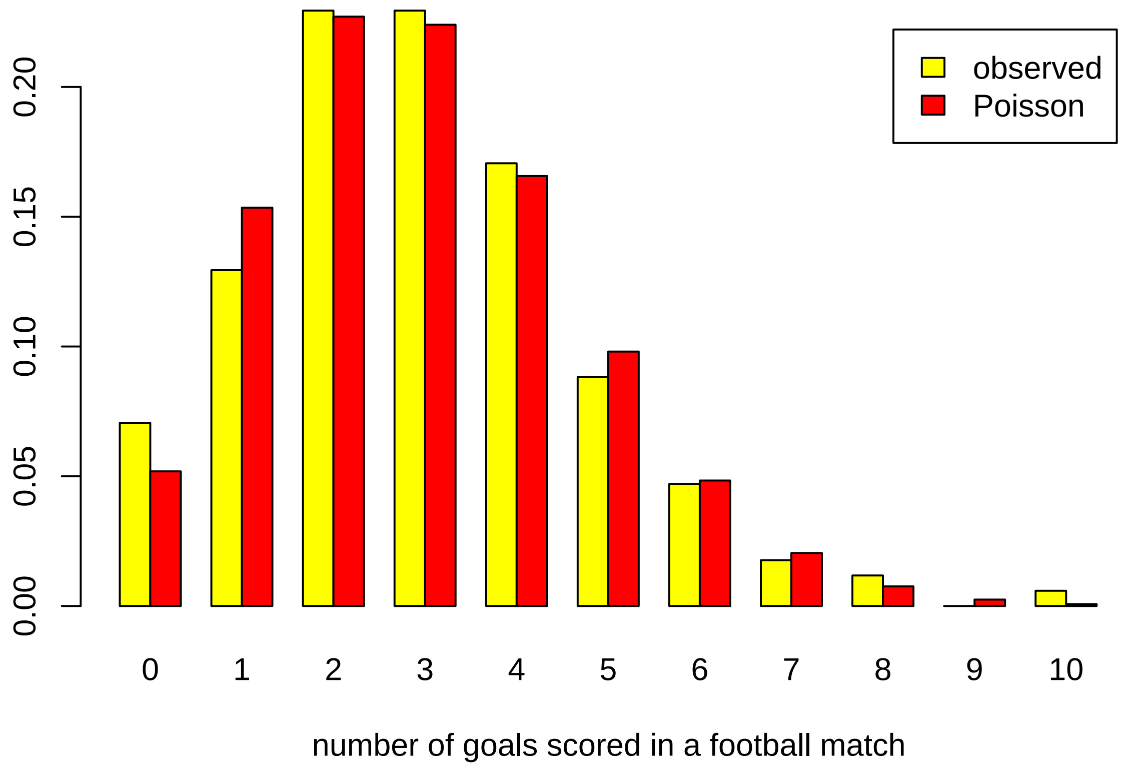
Empirical distribution:



# 2.3 Data and Stochastic Models

## Example 1: The number of goals scored in matches of Beşiktaş.

Empirical distribution and Poisson distribution:



# 2.3 Data and Stochastic Models

Example 1: The number of goals scored in matches of Beşiktaş.

- The close agreement between empirical and theoretical distribution has a clear interpretation.

- Remember that:

$$\lim_{\substack{n\to\infty \\ p\to 0 \\ n\cdot p\to\lambda}} \binom{n}{i} p^i (1-p)^{n-i} = \frac{\lambda^i}{i!} \mathrm{e}^{-\lambda}$$

(This is Poisson's limit theorem.)

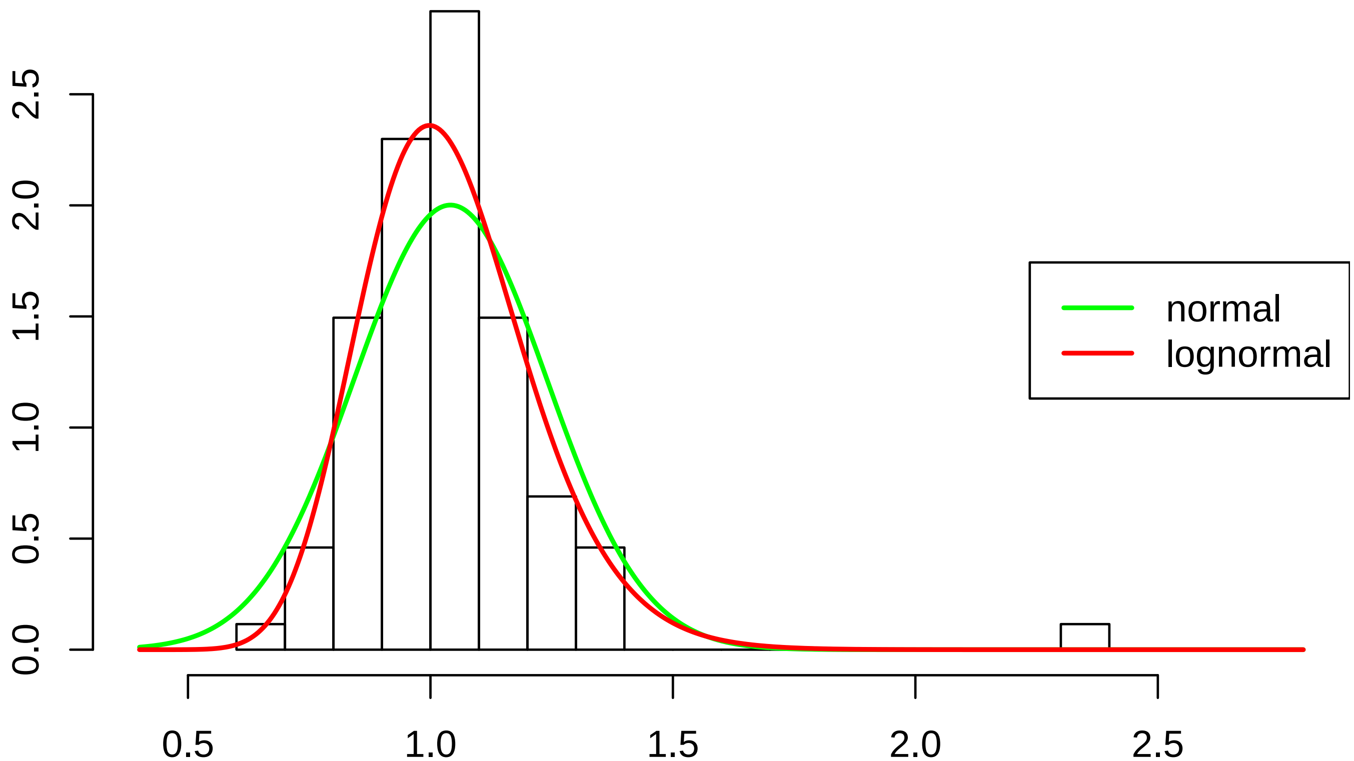
# 2.3 Data and Stochastic Models

Example 2: Gross returns of WTI crude oil.

- We shall now see a histogram of quarterly gross returns (from 1986Q2 through 2007Q4).
- Also shown in the plot:
  - the density of the normal distribution,
  - the density of the lognormal distribution.
- Which one is "better"?

# 2.3 Data and Stochastic Models

Example 2: Gross returns of WTI crude oil.



# 2.3 Data and Stochastic Models

Example 2: Gross returns of WTI crude oil.

Observations and conclusions:

- The lognormal distribution fits somehow better.
  (However, a Kolmogorov-Smirnov test rejects neither null hypothesis.)
- The empirical distribution is right-skewed, and so is the lognormal distribution.
- The outlier (an increase by 130% in 1990Q3!) is not really in line with either distribution.

# 2.3 Data and Stochastic Models

Example 2: Gross returns of WTI crude oil.

Observations and conclusions:

- More important than a good fit:
  Stochastic modeling leads to the lognormal distribution.

- Two roads to lognormally distributed gross returns:
  - the assumption that asset prices follow an exponential Brownian motion,
  - the central limit theorem, applied to daily random factors.

- Like in Example 1, we see the importance of stochastic modeling.

# 2.4 Time Series

An adaptation of the scheme for time series analysis.

**OBSERVATIONS**

- sequence of variables
- observed series
- empirical properties ("stylized facts")

→ (OBSERVATIONS to STOCHASTIC MODEL)

- model identification
- statistical inference

← (STOCHASTIC MODEL to OBSERVATIONS)

- random experiments
- computer simulations

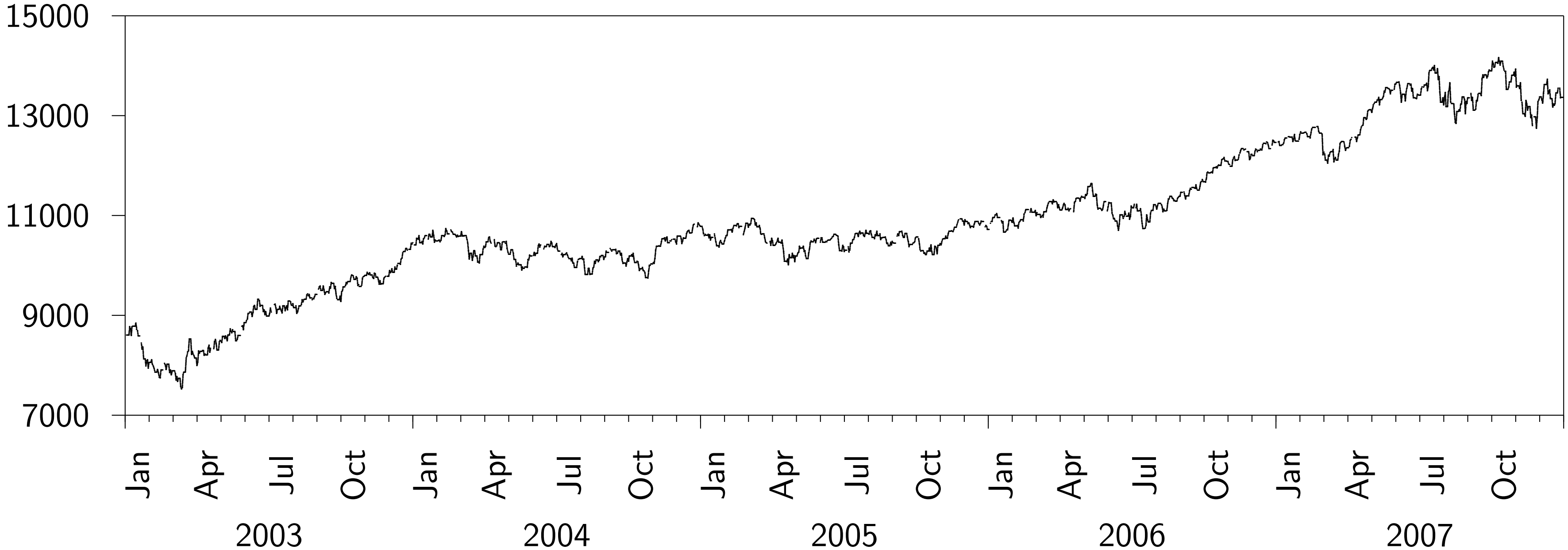
**STOCHASTIC MODEL**

- stochastic process
- trajectory
- theoretical properties

Where could we put *forecasting*?

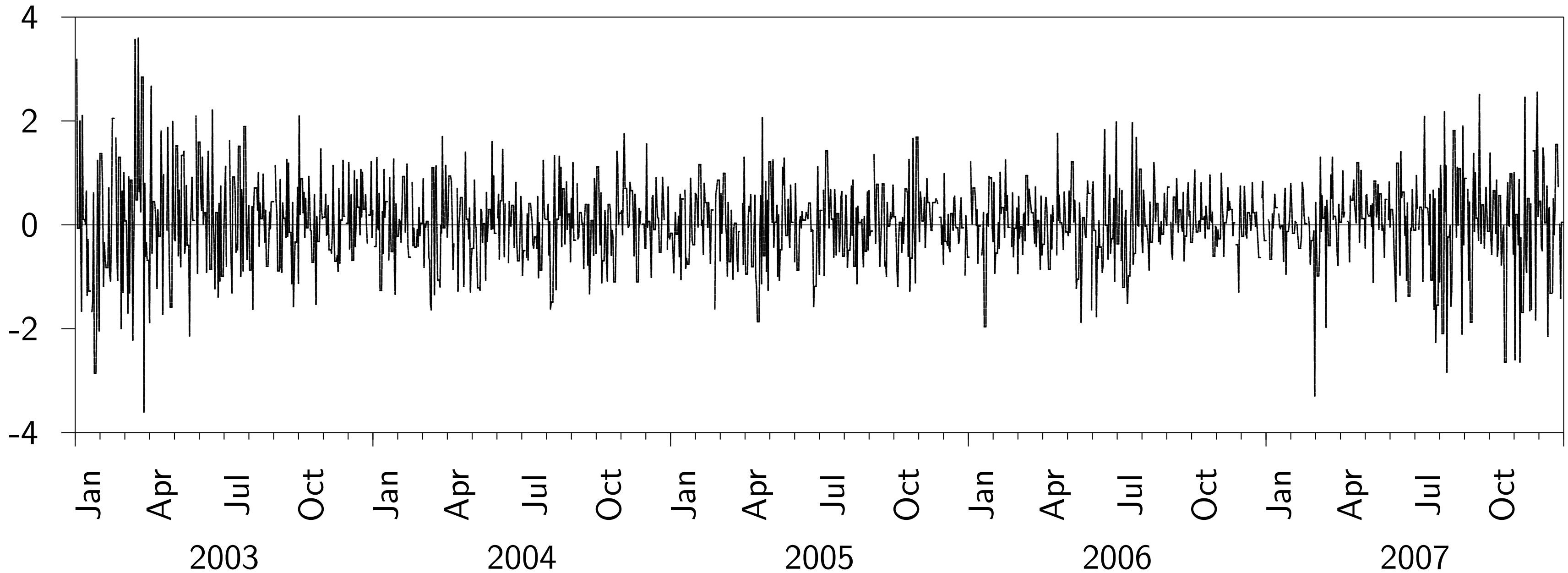
# 2.4 Time Series

Example: DJIA — the level series (daily quotes).



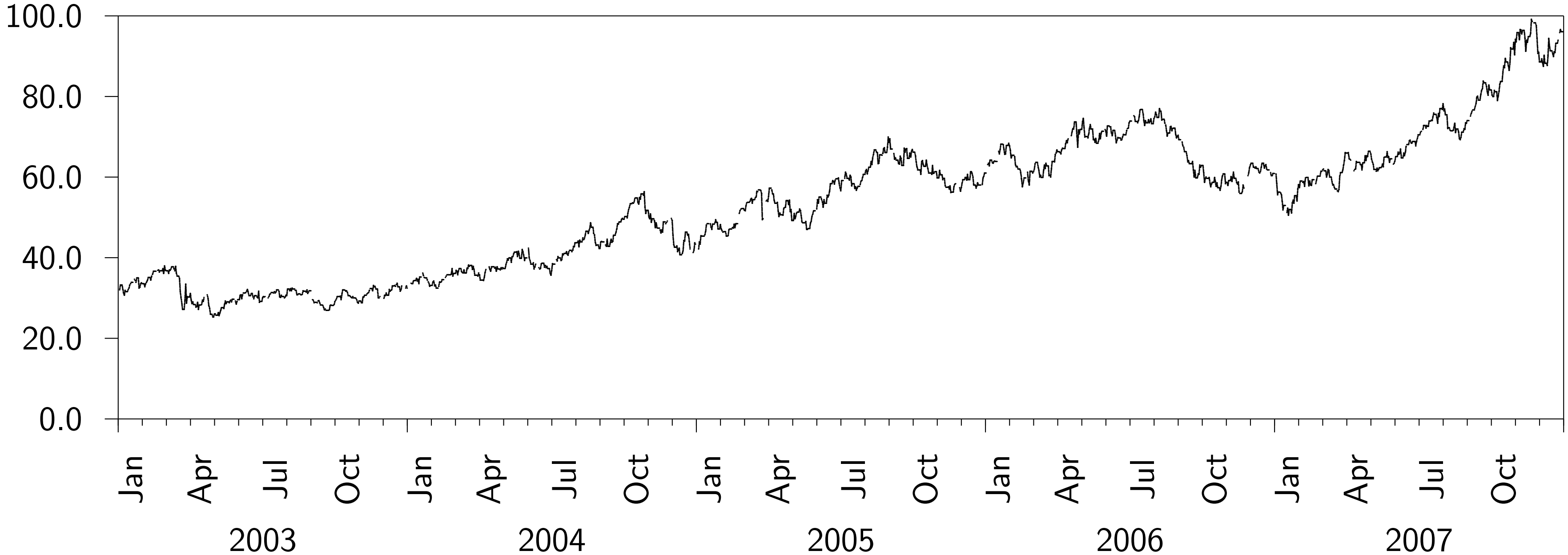
# 2.4 Time Series

Example: DJIA — the series of daily returns.



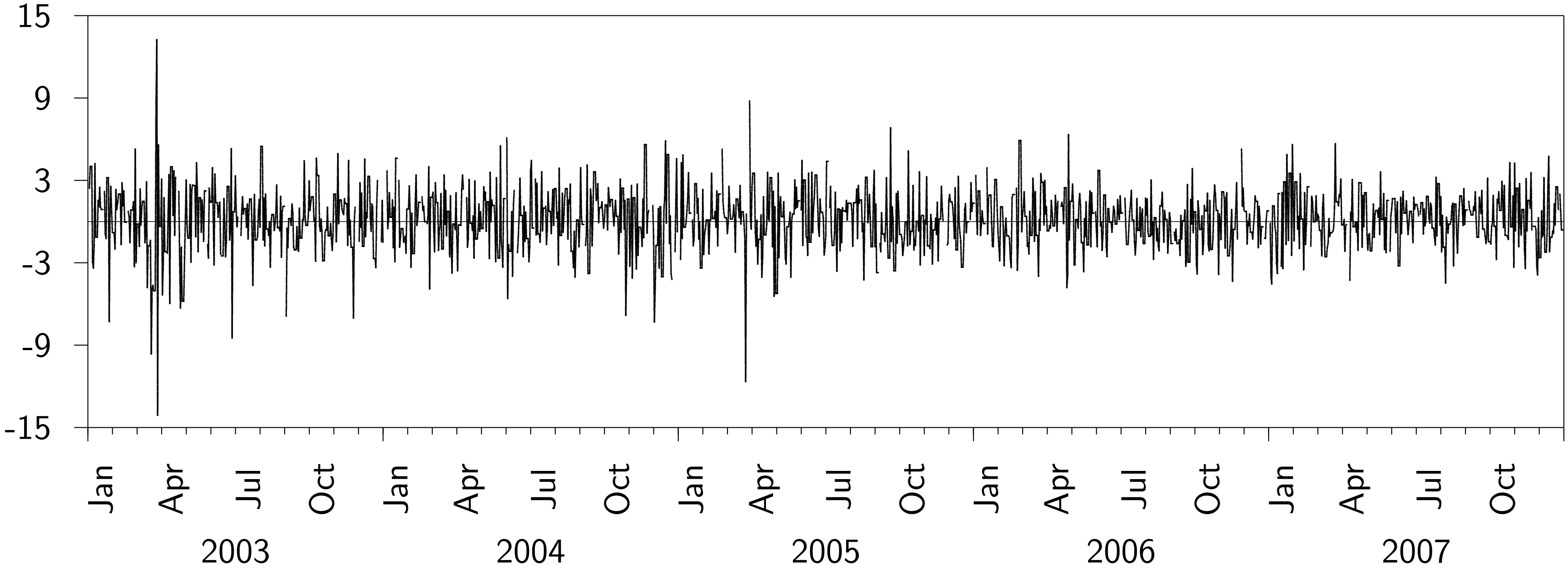
# 2.4 Time Series

Example: WTI — the level series (daily quotes).



# 2.4 Time Series

Example: WTI — the series of daily returns.



# 2.4 Time Series

## Some comments.

- There is a certain form of heteroskedasticity in the return series: High volatility tends to persist for some time.
- This idea leads to (G)ARCH models.
- Good question: What is the relation in the volatilities of the two series?
- Next, we'll shift the perspective.

# 2.5 Empirical Return Distributions

## Distributions as condensed processes.

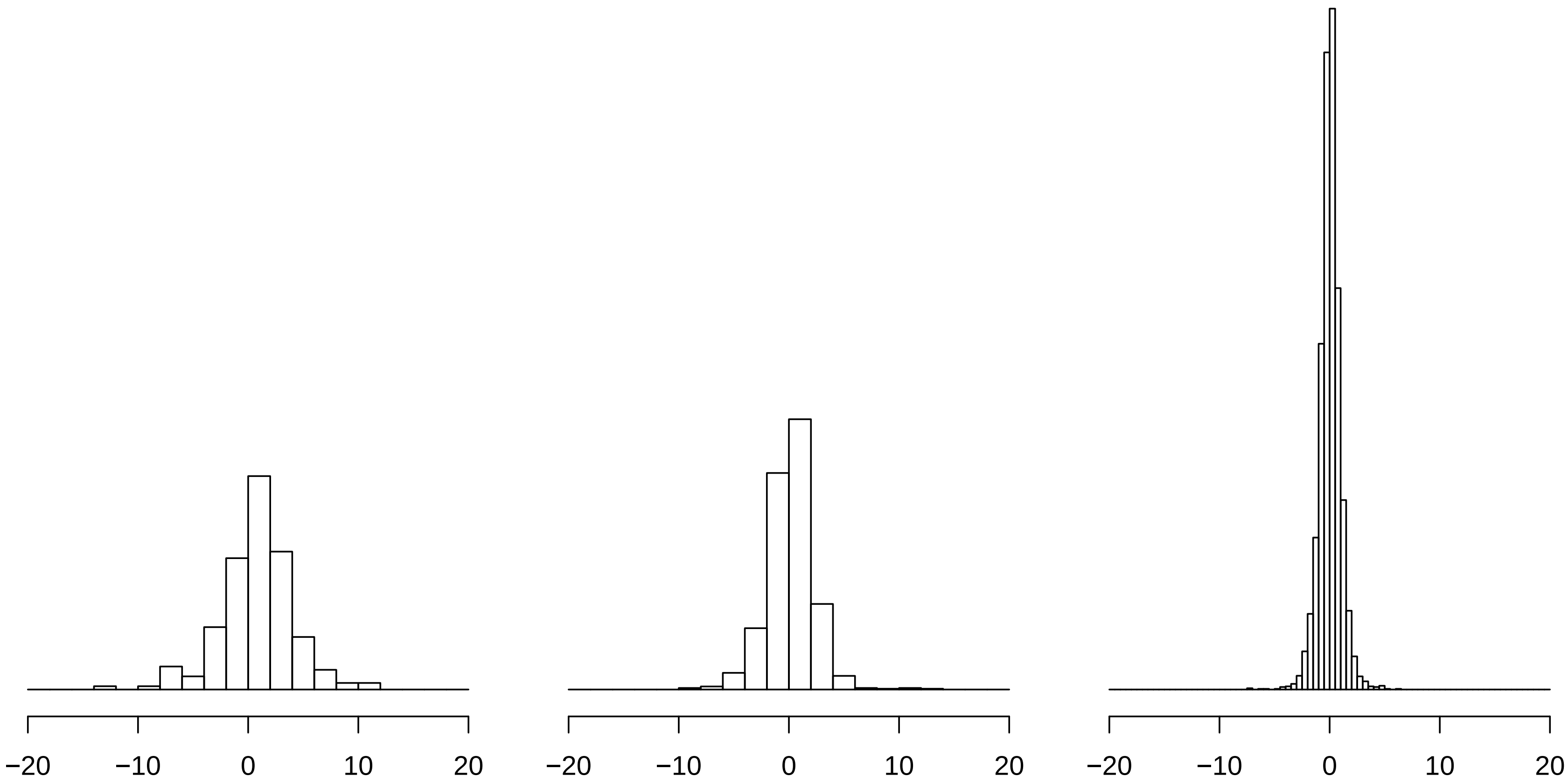
- Compare:
  - time series: When is which value taken on?
  - distribution: How often is a value taken on?

- In this sense, a distribution can be seen as a condensed process.

- For returns, an appropriate display is a histogram.

# 2.5 Empirical Return Distributions

DJIA: monthly, weekly, daily returns (1991-01 – 2008-01).



# 2.6 Does the Normal Distribution Fit?

Once again: data and a stochastic model.

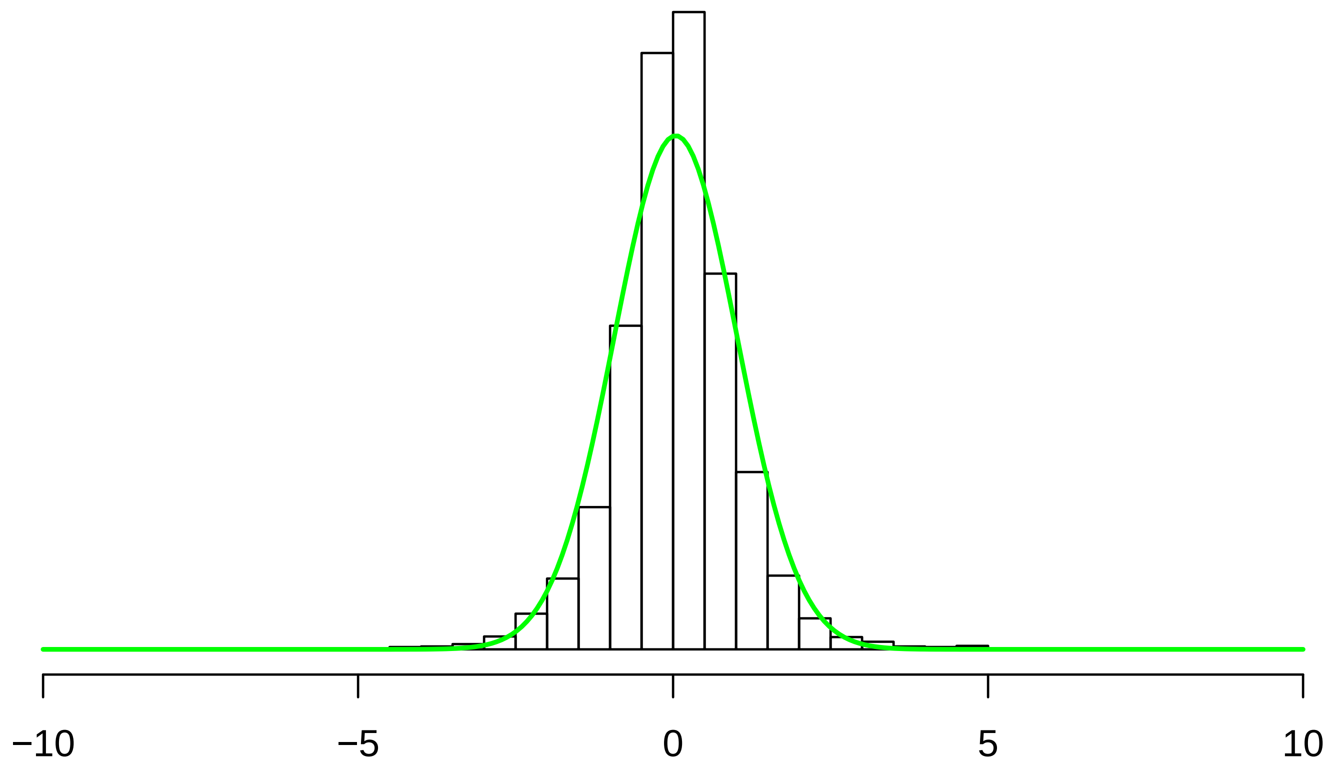
We shall now:

- plot histograms of a return distribution,
- compare the empirical distribution to the normal distribution in terms of
  - a density plot — entire distribution,
  - a density plot — upper tail,
  - a qq-plot,
  - the three sigma-rules.

# 2.6 Does the Normal Distribution Fit?

DJIA: daily returns (1991-01 – 2008-01).

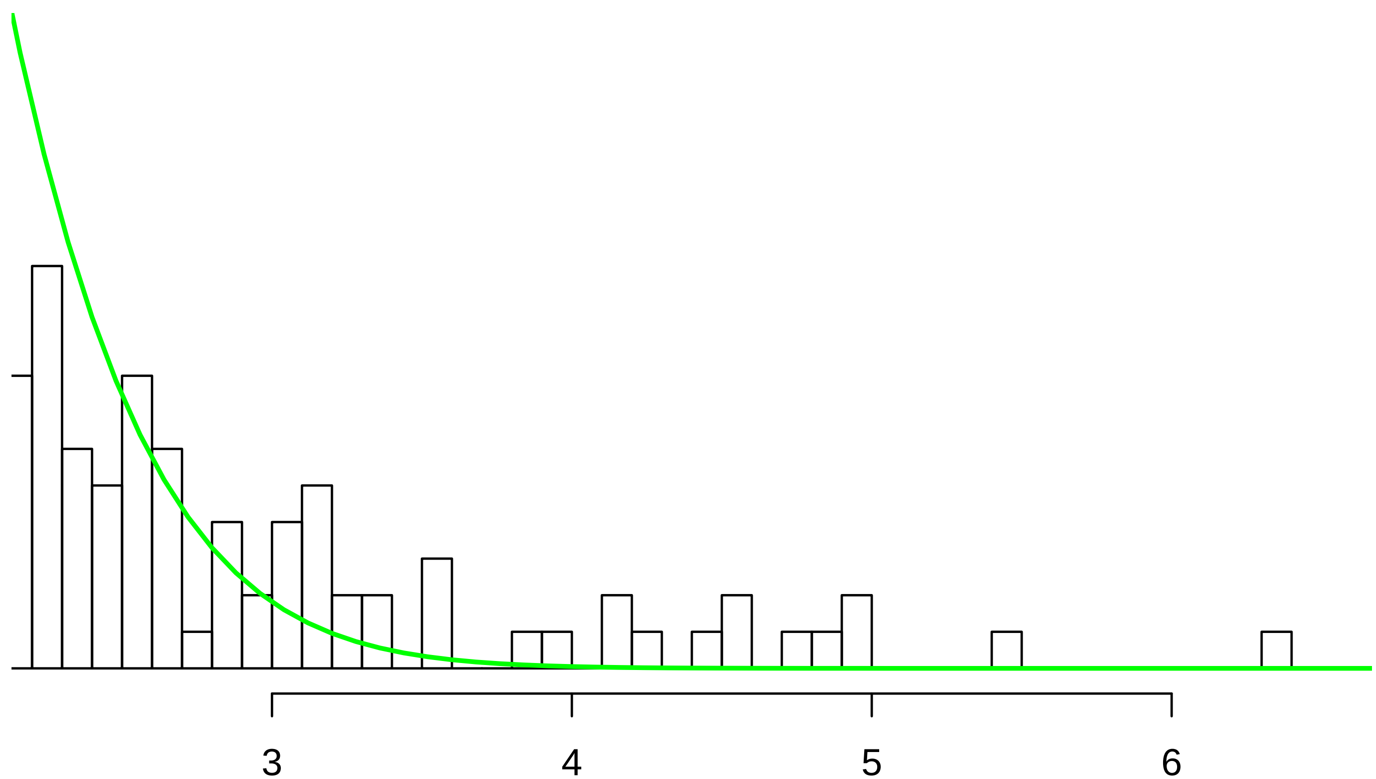
Histogram and normal distribution density:



# 2.6 Does the Normal Distribution Fit?

DJIA: daily returns (1991-01 – 2008-01).

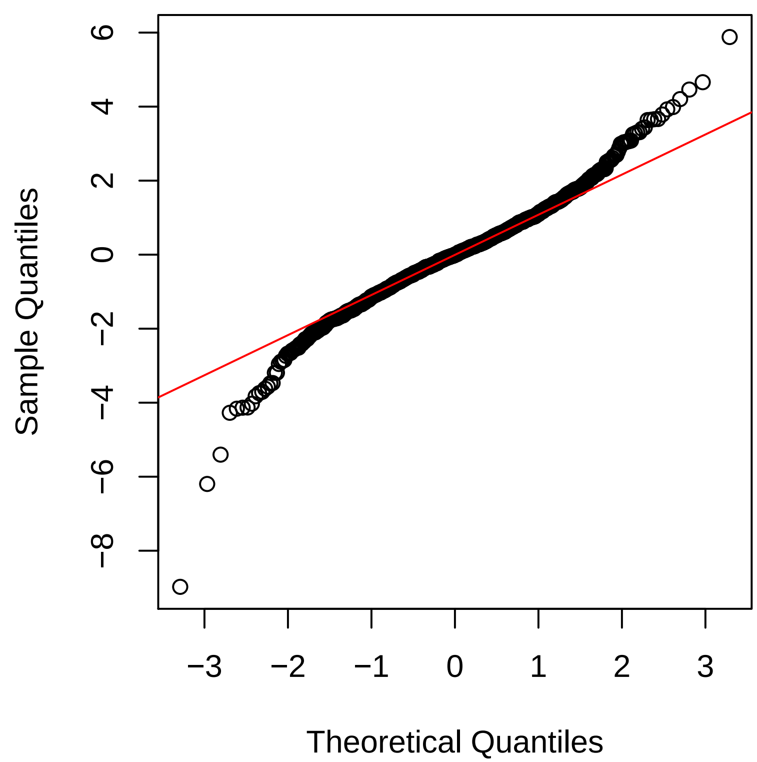
Upper tail in finer resolution:



# 2.6 Does the Normal Distribution Fit?

DJIA: daily returns (1991-01 – 2008-01).

Empirical vs. normal distribution quantiles: qq-plot



# 2.6 Does the Normal Distribution Fit?

## DJIA (1991-01 – 2008-01): Do the sigma-rules hold?

| | monthly | weekly | daily |
|---|---|---|---|
| number of observations | 205 | 889 | 4306 |
| arithmetic mean, $\bar{r}$ | 0.82 | 0.20 | 0.04 |
| variance, $s^2$ | 10.61 | 4.55 | 0.96 |
| standard deviation, $s$ | 3.26 | 2.13 | 0.98 |
| skewness, $\gamma_1$ | -0.34 | 0.12 | -0.14 |
| kurtosis, $\gamma_2$ | 1.72 | 3.47 | 4.56 |
| $[\bar{r} - s, \bar{r} + s]$ | $[\,-2.44\,,\,4.08\,]$ | $[\,-1.93\,,\,2.34\,]$ | $[\,-0.94\,,\,1.02\,]$ |
| observed | 154 | 669 | 3272 |
| expected | 137 | 596 | 2885 |
| $[\bar{r} - 2s, \bar{r} + 2s]$ | $[\,-5.69\,,\,7.33\,]$ | $[\,-4.06\,,\,4.47\,]$ | $[\,-1.92\,,\,2.00\,]$ |
| observed | 190 | 848 | 4074 |
| expected | 195 | 845 | 4091 |
| $[\bar{r} - 3s, \bar{r} + 3s]$ | $[\,-8.95\,,\,10.59\,]$ | $[\,-6.20\,,\,6.60\,]$ | $[\,-2.89\,,\,2.98\,]$ |
| observed | 202 | 878 | 4249 |
| expected | 203 | 880 | 4263 |

# 2.7 Further Properties of Returns

Comparison of daily returns.

| | DJIA | Bovespa | WTI |
|---|---|---|---|
| first day | 1995-01-03 | 1995-01-02 | 1995-01-03 |
| last day | 2008-02-14 | 2008-02-19 | 2008-01-31 |
| observations | 3304 | 3244 | 3281 |
| NAs | 119 | 183 | 132 |
| mean | 0.041 | 0.110 | 0.079 |
| std error | 0.018 | 0.045 | 0.044 |
| var | 1.096 | 5.657 | 5.691 |
| std deviation | 1.047 | 2.378 | 2.386 |
| skewness | −0.175 | 1.173 | −0.123 |
| std error | 0.195 | 0.832 | 0.192 |
| kurtosis | 4.112 | 19.843 | 4.311 |
| std error | 0.722 | 9.267 | 0.821 |

# 2.7 Further Properties of Returns

Comparison of daily returns.

| | DJIA | Bovespa | WTI |
|---|---|---|---|
| first day | 1995-01-03 | 1995-01-02 | 1995-01-03 |
| last day | 2008-02-14 | 2008-02-19 | 2008-01-31 |
| min | −7.184 | −15.809 | −15.711 |
| lower quartile | −0.497 | −1.086 | −1.222 |
| median | 0.056 | 0.144 | 0.116 |
| upper quartile | 0.603 | 1.360 | 1.438 |
| max | 6.348 | 33.419 | 16.629 |
| day of min | 1997-10-27 | 1998-09-10 | 2001-09-24 |
| day of max | 2002-07-24 | 1999-01-15 | 1998-04-27 |

# 2.8 Summary and Outlook

- Empirical return distributions have heavy tails.
- The normal distribution does not really fit.
- Heavy tails are caused by the special form of heteroskedasticity.

# 2.8 Summary and Outlook

Different stochastic models cope with this situation in different ways:

- Brownian motion ignores heavy tails.
- GARCH processes are dynamic volatility models.
- The generalized Pareto distribution models tails.

The present course shows some of these approaches in action.