

**EC 613:
Advanced Topics in Financial Econometrics**

**FEC 514:
Applications in Financial Modeling**

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 İSTANBUL BİLGİ ÜNİVERSİTESİ

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- R files used for this course are available upon request.



Chapter 2:

Analyzing Price Changes: Some Aspects



2.1 Introduction

The starting point.

- Typical starting point: sequence $(P_t)_t$ of price quotes
- Often more interesting: price changes, i.e. returns

simple return: $R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ or $\frac{P_t - P_{t-1}}{P_{t-1}} \cdot 100\%$

log return: $r_t = \ln \frac{P_t}{P_{t-1}}$

gross return: $1 + R_t = \frac{P_t}{P_{t-1}}$



2.1 Introduction

The starting point.

- The analysis of (R_t) or (r_t) or $(1 + R_t)$ can then:
 - either neglect time
(good for understanding the average behaviour of the series)
 - or take time explicitly into account
(good for understanding the conditional behaviour of the series)
- In any case, we need the methods of statistics!
- A very brief review of the objectives of statistical analysis is therefore in order.



2.2 Statistics: Reasoning With Numbers

The goals of descriptive and inductive statistics.

The goal of. . .

- . . . descriptive statistics is: Describe, summarize, display given data (data reduction!).
- . . . inductive statistics is: Draw conclusions from data (observations) to more general principles.

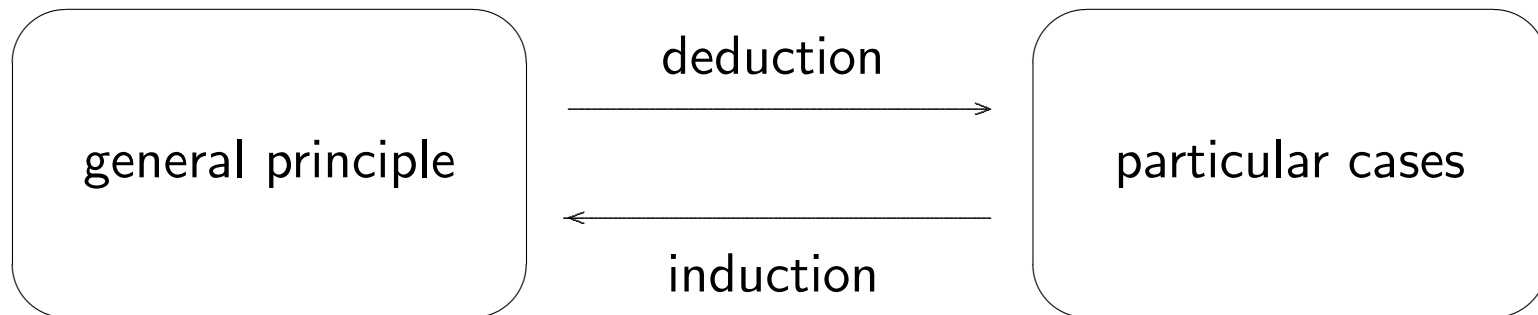
The process of drawing conclusions is called statistical inference.



2.2 Statistics: Reasoning With Numbers

Conclusions.

There are two kinds of conclusions:



- In the context of inductive statistics, the “particular cases” are observed data (sample data).
- The “general principle” is a probability distribution, characterizing the entire population.



2.2 Statistics: Reasoning With Numbers

Inductive statistics.

- The paradigm of inductive statistics is:

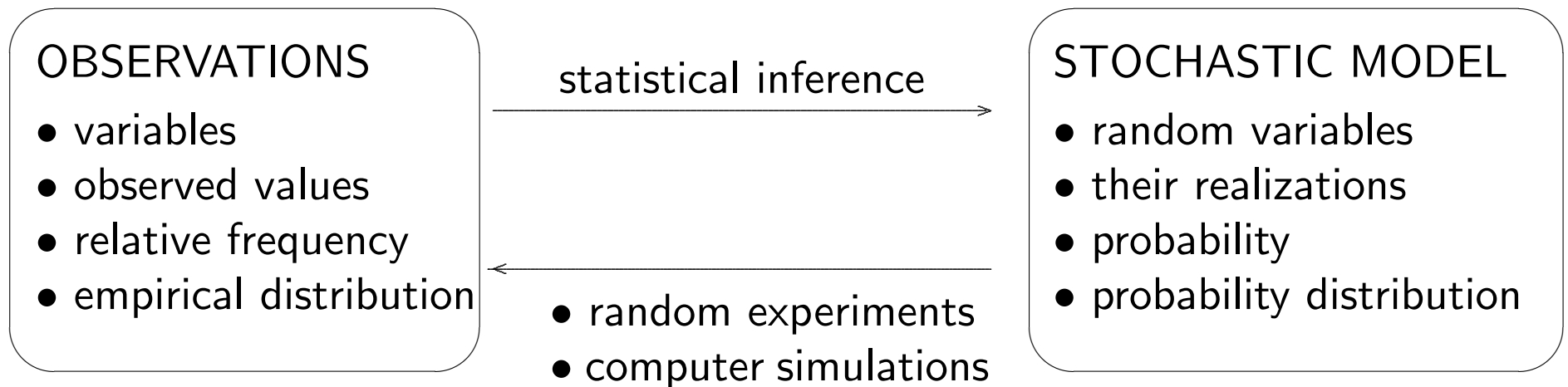
Regard the observations as the outcome of a random experiment, that is, as being produced by a stochastic model.

- Stochastic model: a mathematical model on the basis of probability.
- The object of research is then the stochastic model, rather than the observations!



2.2 Statistics: Reasoning With Numbers

Observations and stochastic models: analogies and their relation.



2.3 Data and Stochastic Models

The role of stochastic modeling: two examples.

Now, for two simple examples, we shall:

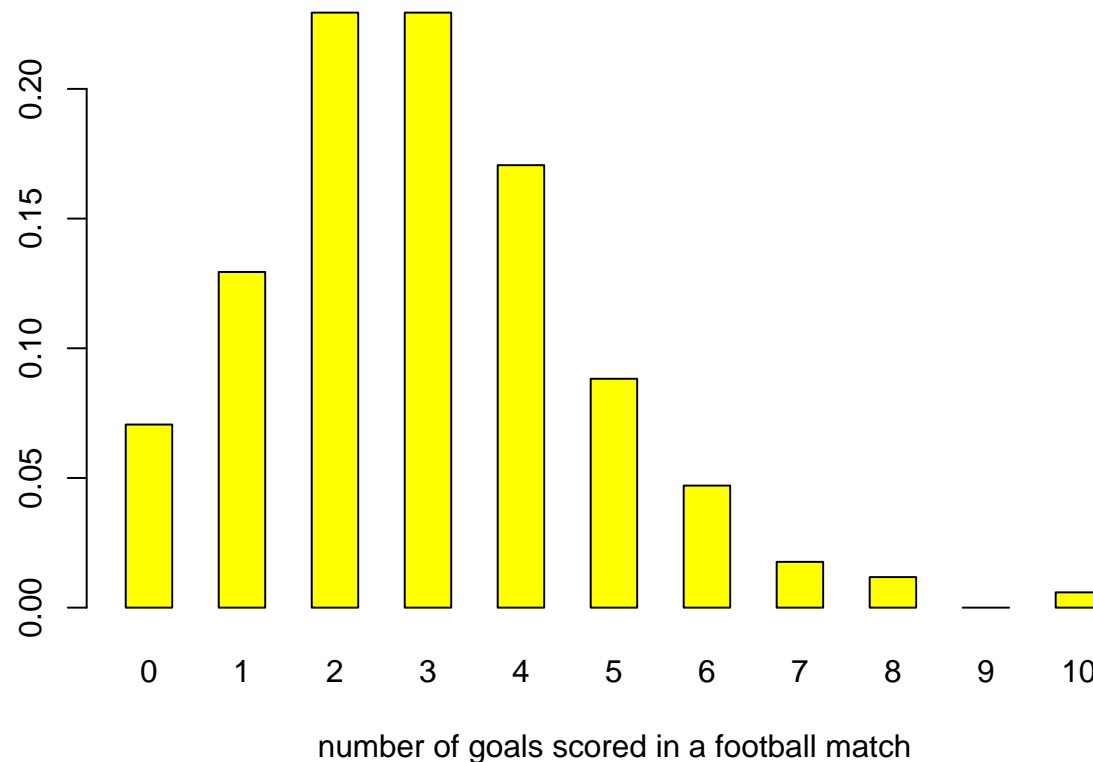
- see the empirical evidence in the form of a chart,
- add a plot of a suitable stochastic model to the chart,
- discuss briefly what the stochastic model can tell us.



2.3 Data and Stochastic Models

Example 1: The number of goals scored in matches of Beşiktaş.

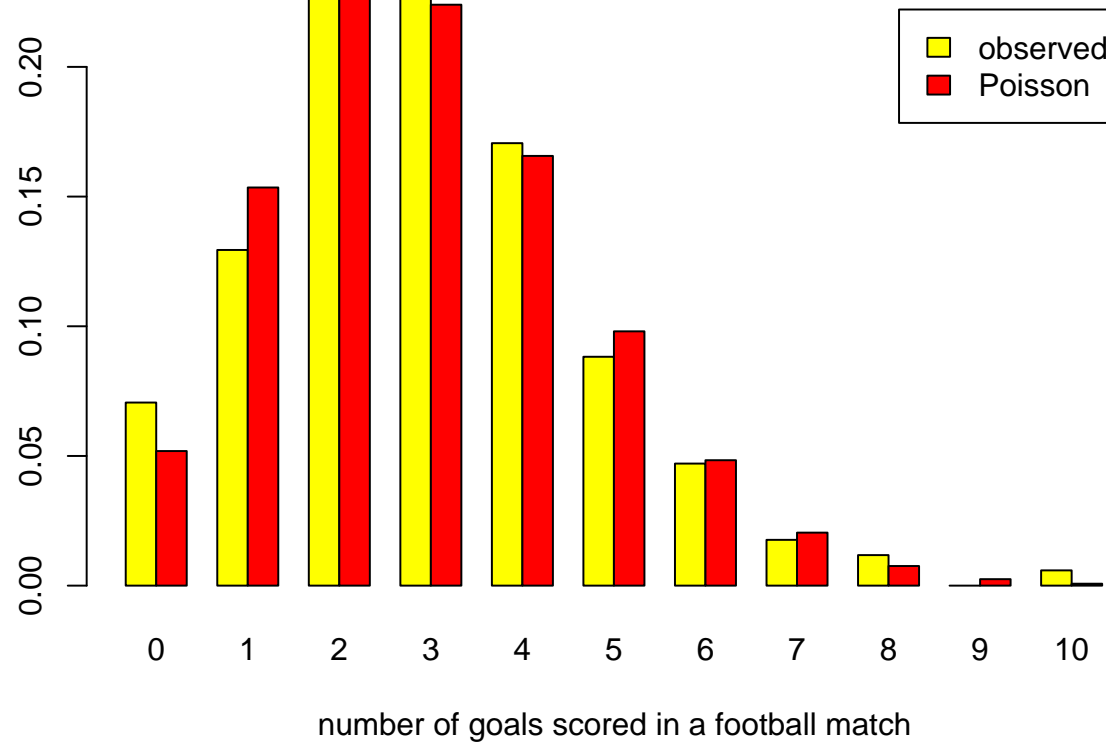
Empirical distribution:



2.3 Data and Stochastic Models

Example 1: The number of goals scored in matches of Beşiktaş.

Empirical distribution and Poisson distribution:



2.3 Data and Stochastic Models

Example 1: The number of goals scored in matches of Beşiktaş.

- The close agreement between empirical and theoretical distribution has a clear interpretation.
- Remember that:

$$\lim_{\substack{n \rightarrow \infty \\ p \rightarrow 0 \\ n \cdot p \rightarrow \lambda}} \binom{n}{i} p^i (1-p)^{n-i} = \frac{\lambda^i}{i!} e^{-\lambda}$$

(This is Poisson's limit theorem.)



2.3 Data and Stochastic Models

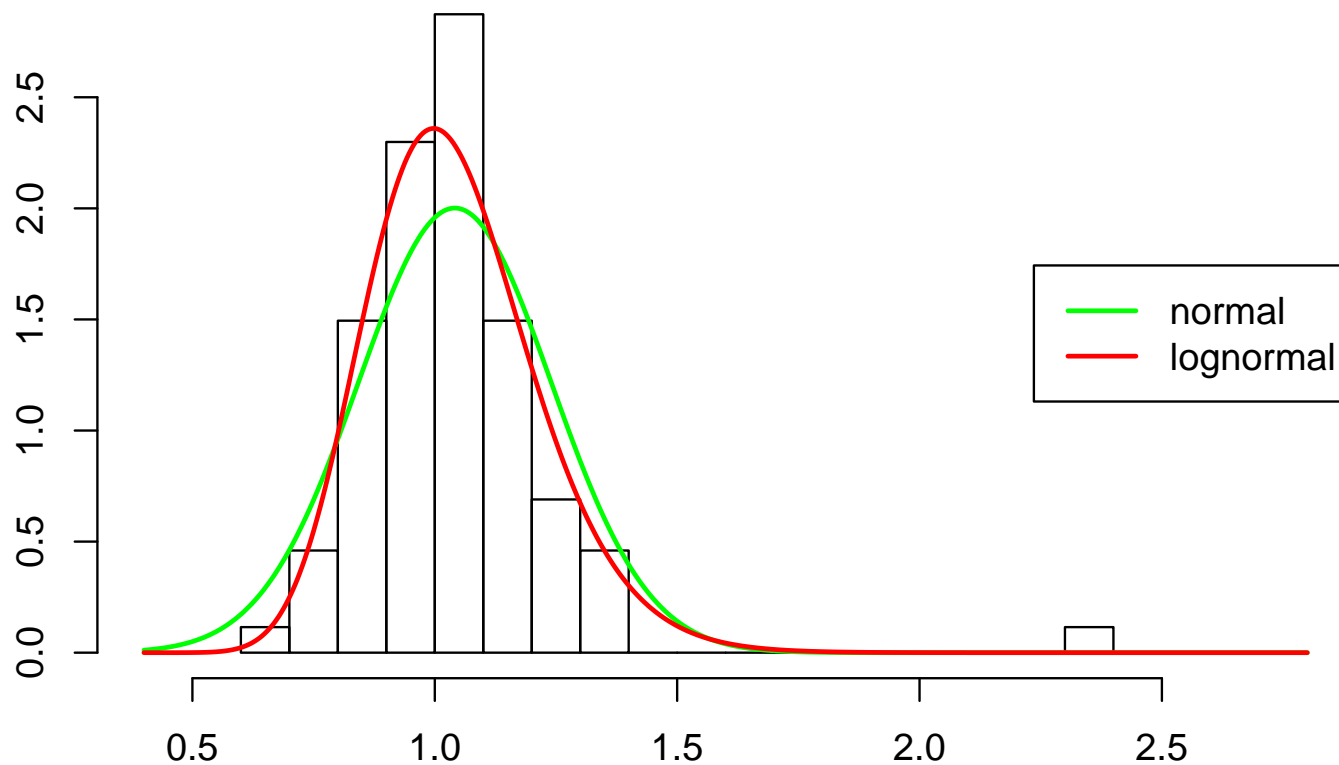
Example 2: Gross returns of WTI crude oil.

- We shall now see a histogram of quarterly gross returns (from 1986Q2 through 2007Q4).
- Also shown in the plot:
 - the density of the normal distribution,
 - the density of the lognormal distribution.
- Which one is “better”?



2.3 Data and Stochastic Models

Example 2: Gross returns of WTI crude oil.



2.3 Data and Stochastic Models

Example 2: Gross returns of WTI crude oil.

Observations and conclusions:

- The lognormal distribution fits somehow better. (However, a Kolmogorov-Smirnov test rejects neither null hypothesis.)
- The empirical distribution is right-skewed, and so is the lognormal distribution.
- The outlier (an increase by 130% in 1990Q3!) is not really in line with either distribution.



2.3 Data and Stochastic Models

Example 2: Gross returns of WTI crude oil.

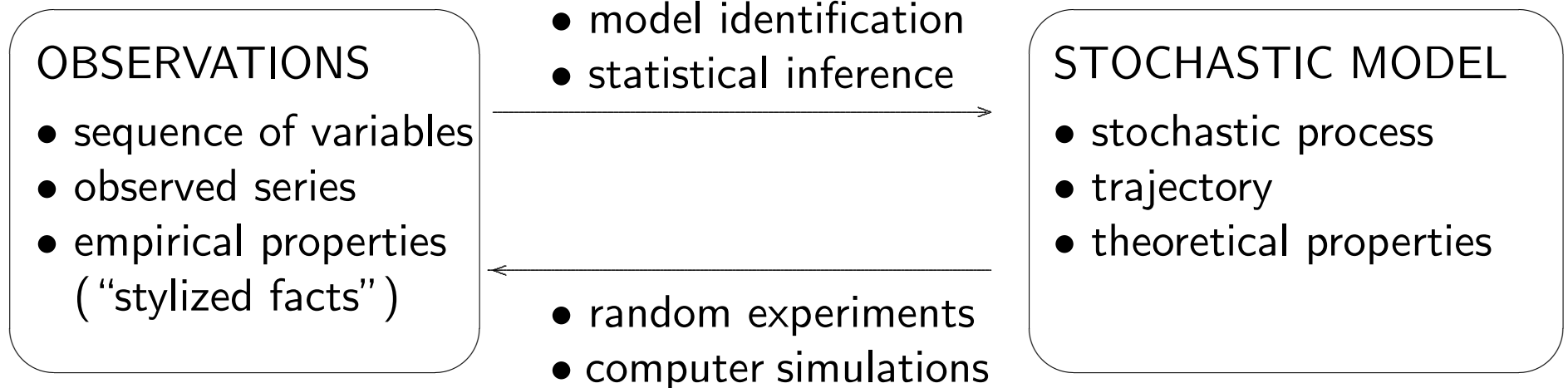
Observations and conclusions:

- More important than a good fit:
Stochastic modeling leads to the lognormal distribution.
- Two roads to lognormally distributed gross returns:
 - the assumption that asset prices follow an exponential Brownian motion,
 - the central limit theorem, applied to daily random factors.
- Like in Example 1, we see the importance of stochastic modeling.



2.4 Time Series

An adaptation of the scheme for time series analysis.

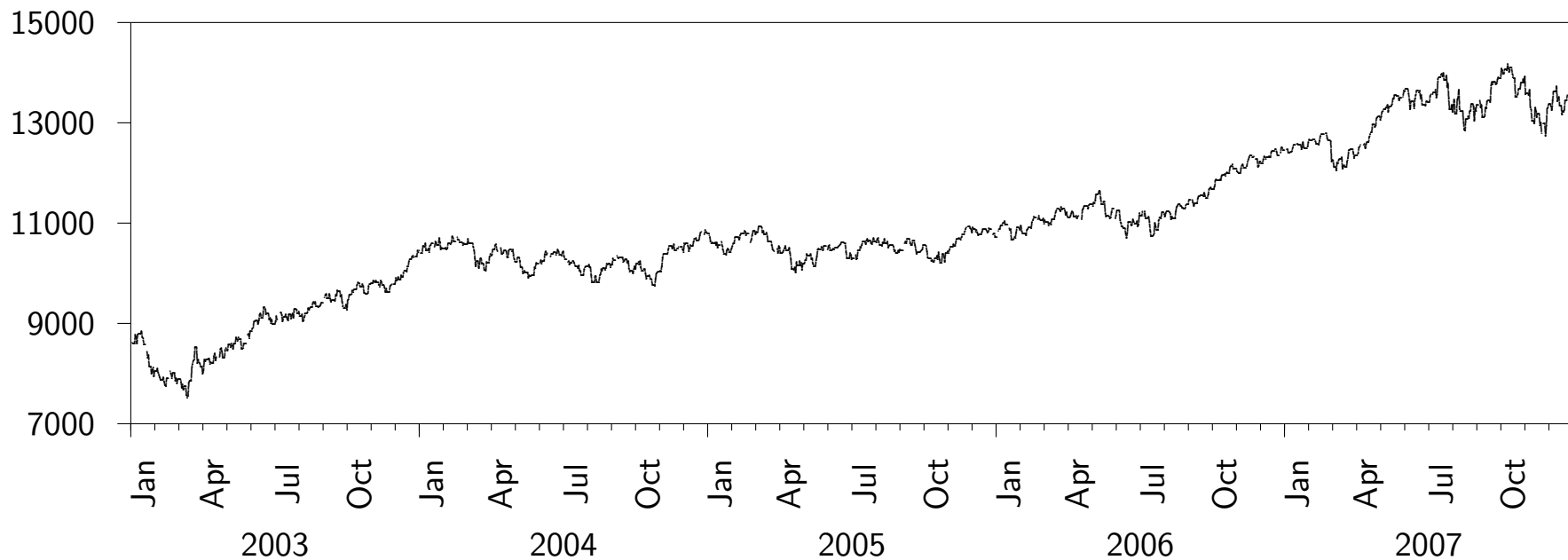


Where could we put *forecasting*?



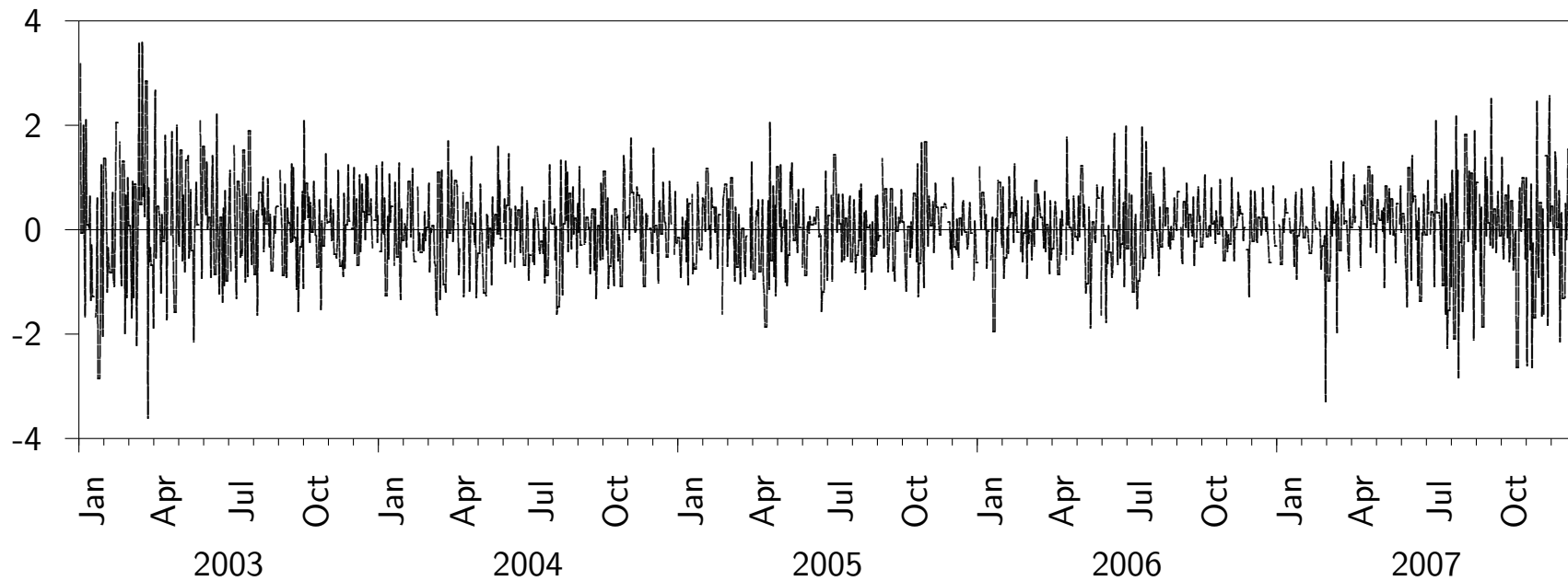
2.4 Time Series

Example: DJIA — the level series (daily quotes).



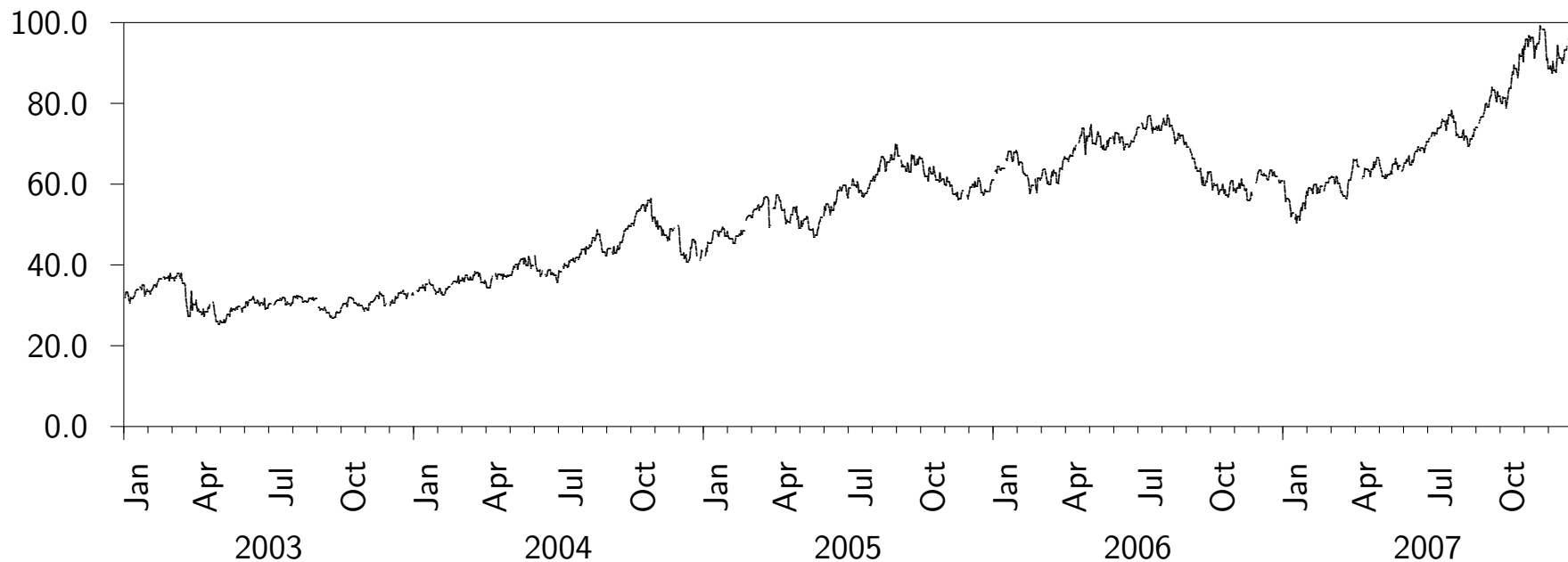
2.4 Time Series

Example: DJIA — the series of daily returns.



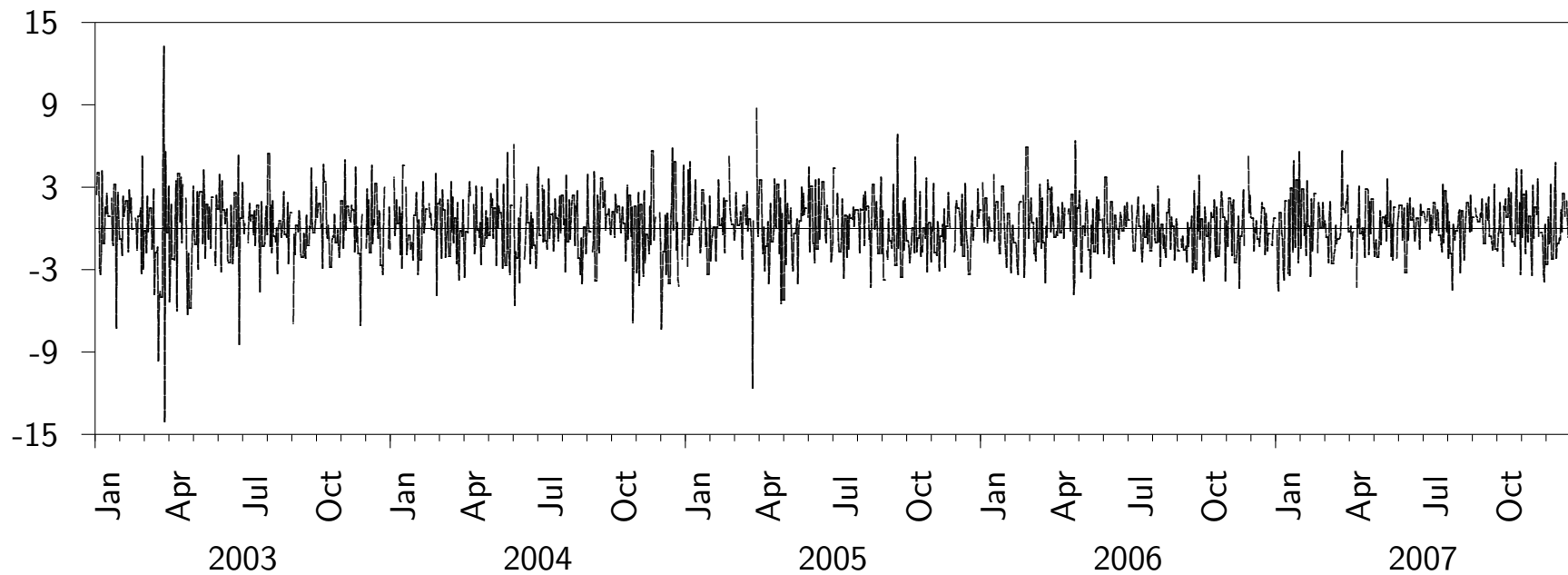
2.4 Time Series

Example: WTI — the level series (daily quotes).



2.4 Time Series

Example: WTI — the series of daily returns.



2.4 Time Series

Some comments.

- There is a certain form of heteroskedasticity in the return series: High volatility tends to persist for some time.
- This idea leads to (G)ARCH models.
- Good question: What is the relation in the volatilities of the two series?
- Next, we'll shift the perspective.



2.5 Empirical Return Distributions

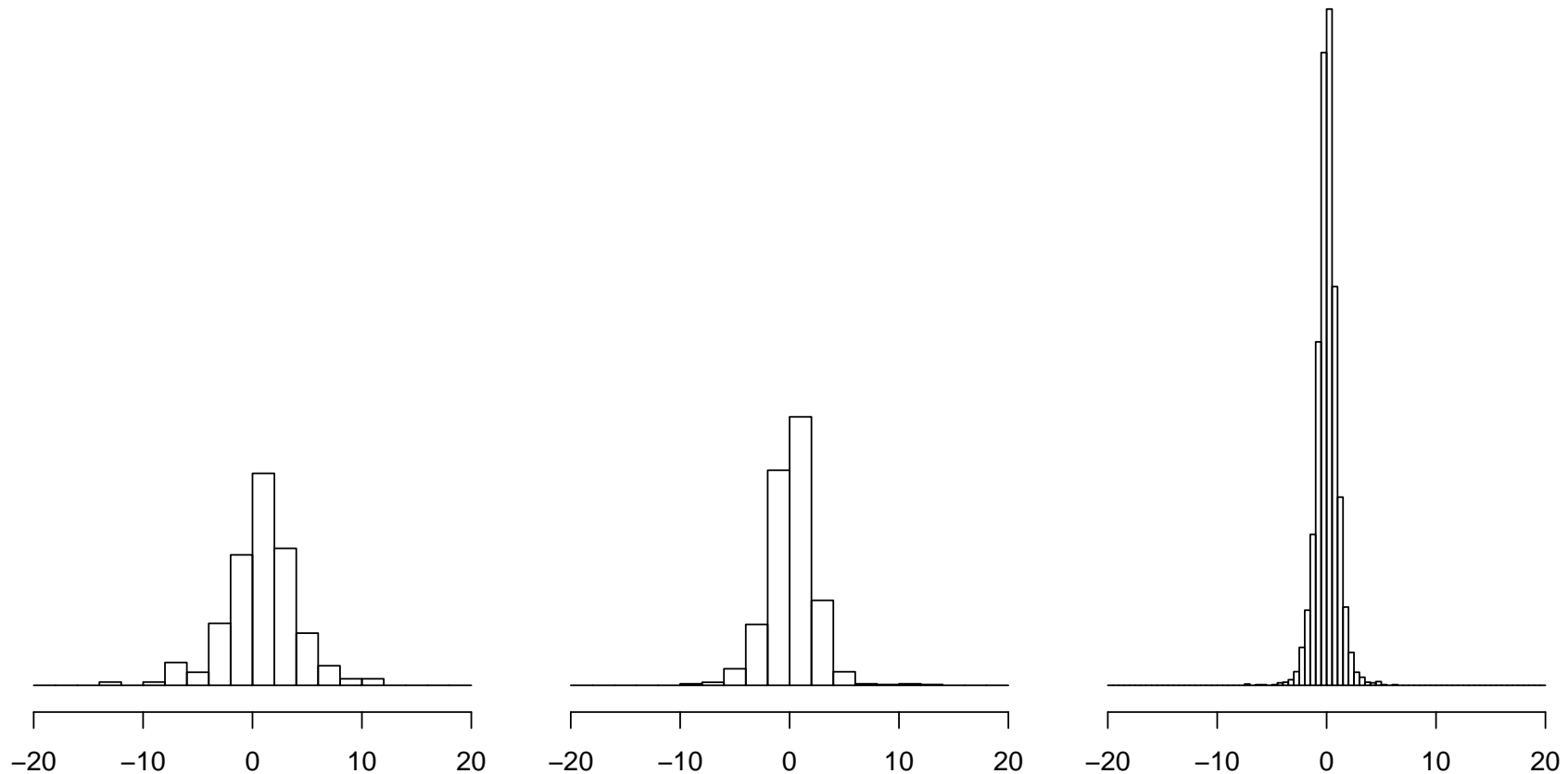
Distributions as condensed processes.

- Compare:
 - time series: When is which value taken on?
 - distribution: How often is a value taken on?
- In this sense, a distribution can be seen as a condensed process.
- For returns, an appropriate display is a histogram.



2.5 Empirical Return Distributions

DJIA: monthly, weekly, daily returns (1991-01 – 2008-01).



2.6 Does the Normal Distribution Fit?

Once again: data and a stochastic model.

We shall now:

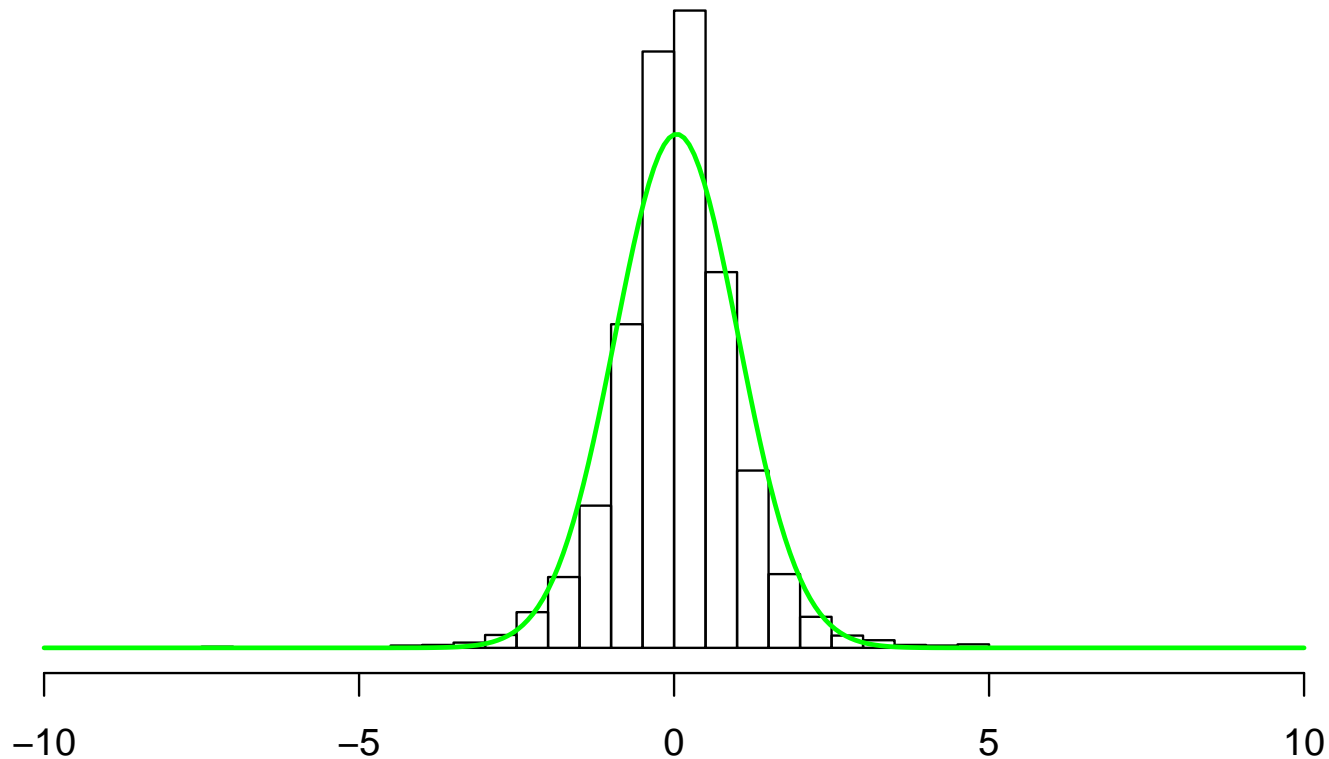
- plot histograms of a return distribution,
- compare the empirical distribution to the normal distribution in terms of
 - a density plot — entire distribution,
 - a density plot — upper tail,
 - a qq-plot,
 - the three sigma-rules.



2.6 Does the Normal Distribution Fit?

DJIA: daily returns (1991-01 – 2008-01).

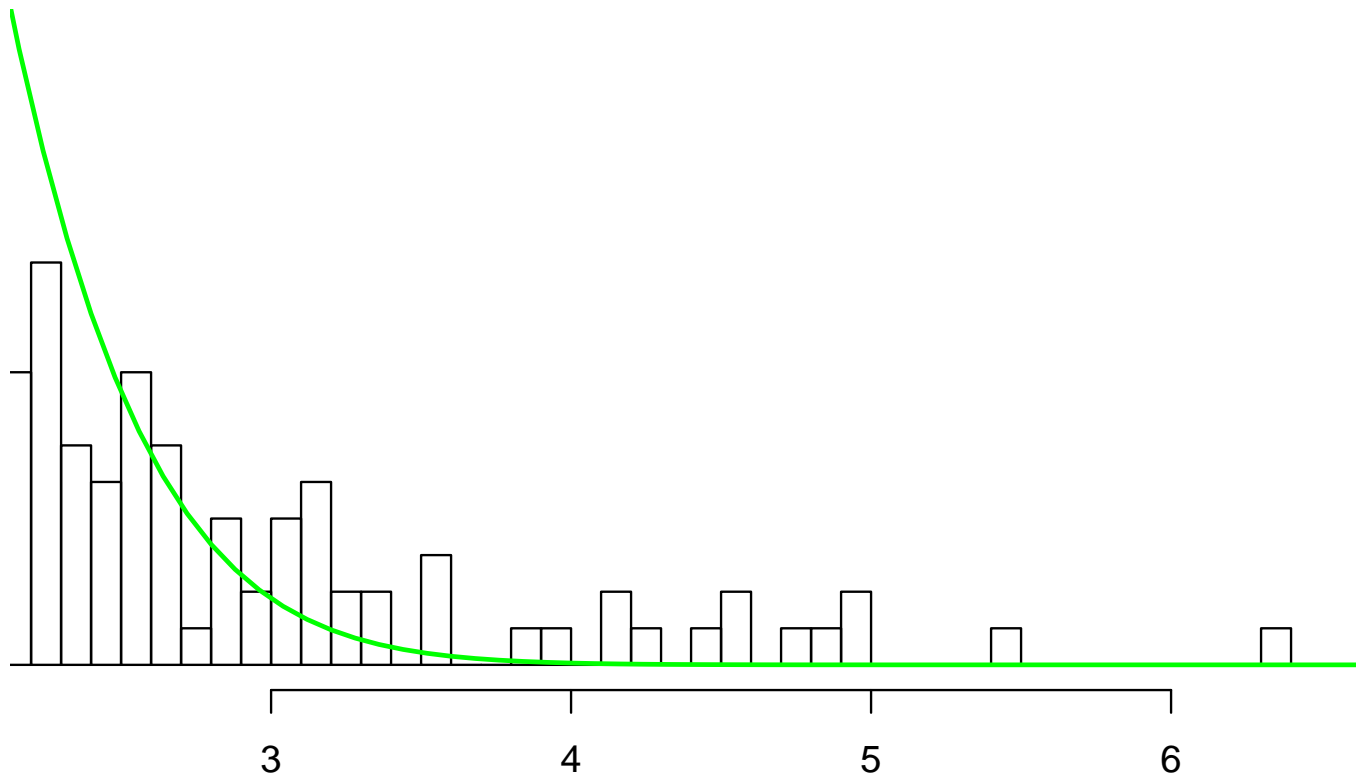
Histogram and normal distribution density:



2.6 Does the Normal Distribution Fit?

DJIA: daily returns (1991-01 – 2008-01).

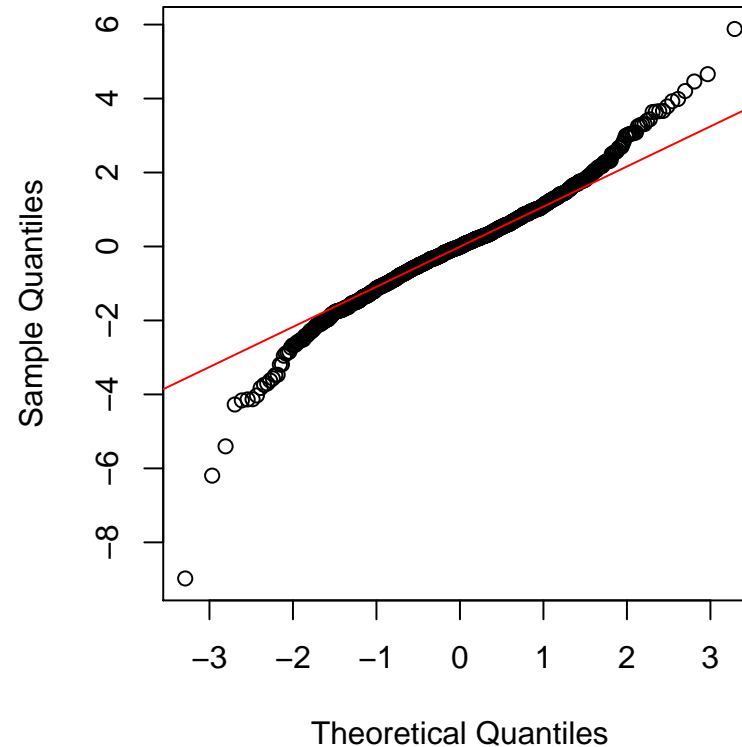
Upper tail in finer resolution:



2.6 Does the Normal Distribution Fit?

DJIA: daily returns (1991-01 – 2008-01).

Empirical vs. normal distribution quantiles: qq-plot



2.6 Does the Normal Distribution Fit?

DJIA (1991-01 – 2008-01): Do the sigma-rules hold?

	monthly	weekly	daily
number of observations	205	889	4306
arithmetic mean, \bar{r}	0.82	0.20	0.04
variance, s^2	10.61	4.55	0.96
standard deviation, s	3.26	2.13	0.98
skewness, γ_1	-0.34	0.12	-0.14
kurtosis, γ_2	1.72	3.47	4.56
$[\bar{r} - s, \bar{r} + s]$	$[-2.44, 4.08]$	$[-1.93, 2.34]$	$[-0.94, 1.02]$
observed	154	669	3272
expected	137	596	2885
$[\bar{r} - 2s, \bar{r} + 2s]$	$[-5.69, 7.33]$	$[-4.06, 4.47]$	$[-1.92, 2.00]$
observed	190	848	4074
expected	195	845	4091
$[\bar{r} - 3s, \bar{r} + 3s]$	$[-8.95, 10.59]$	$[-6.20, 6.60]$	$[-2.89, 2.98]$
observed	202	878	4249
expected	203	880	4263



2.7 Further Properties of Returns

Comparison of daily returns.

	DJIA	Bovespa	WTI
first day	1995-01-03	1995-01-02	1995-01-03
last day	2008-02-14	2008-02-19	2008-01-31
observations	3304	3244	3281
NAs	119	183	132
mean	0.041	0.110	0.079
std error	0.018	0.045	0.044
var	1.096	5.657	5.691
std deviation	1.047	2.378	2.386
skewness	-0.175	1.173	-0.123
std error	0.195	0.832	0.192
kurtosis	4.112	19.843	4.311
std error	0.722	9.267	0.821



2.7 Further Properties of Returns

Comparison of daily returns.

	DJIA	Bovespa	WTI
first day	1995-01-03	1995-01-02	1995-01-03
last day	2008-02-14	2008-02-19	2008-01-31
min	-7.184	-15.809	-15.711
lower quartile	-0.497	-1.086	-1.222
median	0.056	0.144	0.116
upper quartile	0.603	1.360	1.438
max	6.348	33.419	16.629
day of min	1997-10-27	1998-09-10	2001-09-24
day of max	2002-07-24	1999-01-15	1998-04-27



2.8 Summary and Outlook

- Empirical return distributions have heavy tails.
- The normal distribution does not really fit.
- Heavy tails are caused by the special form of heteroskedasticity.



2.8 Summary and Outlook

Different stochastic models cope with this situation in different ways:

- Brownian motion ignores heavy tails.
- GARCH processes are dynamic volatility models.
- The generalized Pareto distribution models tails.

The present course shows some of these approaches in action.

