Bus 701: Advanced Statistics

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- The slides were produced using LaTeX and R (the R project; www.R-project.org) on a GNU/Linux system.

- R files used for this course are available upon request.

Chapter 3:

Displaying Univariate Data

3.1 Frequency Distributions

The notion of frequency distribution.

- So far, we were concerned with the structure of data.
- To obtain insight into a heap of data, we need to look at the frequency distribution of the variable in question.
- The question "How often is each value taken on?" leads to the notion of frequency distribution.

3.1 Frequency Distributions

Frequencies.

Let our observations of a variable X be given as

$$x_1, x_2, \ldots, x_n.$$

Further, let

$$a_1, a_2, \ldots, a_k$$

be the values which appear among the observations.
Then:

$h(a_j)$	$=$	# observations equal to a_j:	**absolute frequency** of a_j
$f(a_j)$	$=$	$h(a_j)/n$:	**relative frequency** of a_j

A list of the a_j, together with their frequencies, is called the (empirical) distribution of X.

3.1 Frequency Distributions

Example:

Observations of $X = gender$ from Example 1:

m, m, m, m, f, f, f, f, f, f, m, m, m, m, m, m, f, f, f, f, f, f, f, f, m, f, m, f, f, m, m, f, m, f, m, m, m, f, m, m, f, f, f, f, f, f, m, m, m, f, f, f, m, m

Distribution of X among the 54 students:

a_j	h_j	f_j
$a_1 = f$	29	29/54
$a_2 = m$	25	25/54
$\sum$	54	1

3.1 Frequency Distributions

Which graphical display is appropriate?

This depends on the scaling of the variable. Among others:

- categorical variable: pie chart, bar chart
- discrete metric variable: bar chart
- continuous metric variable: stemplot, histogram

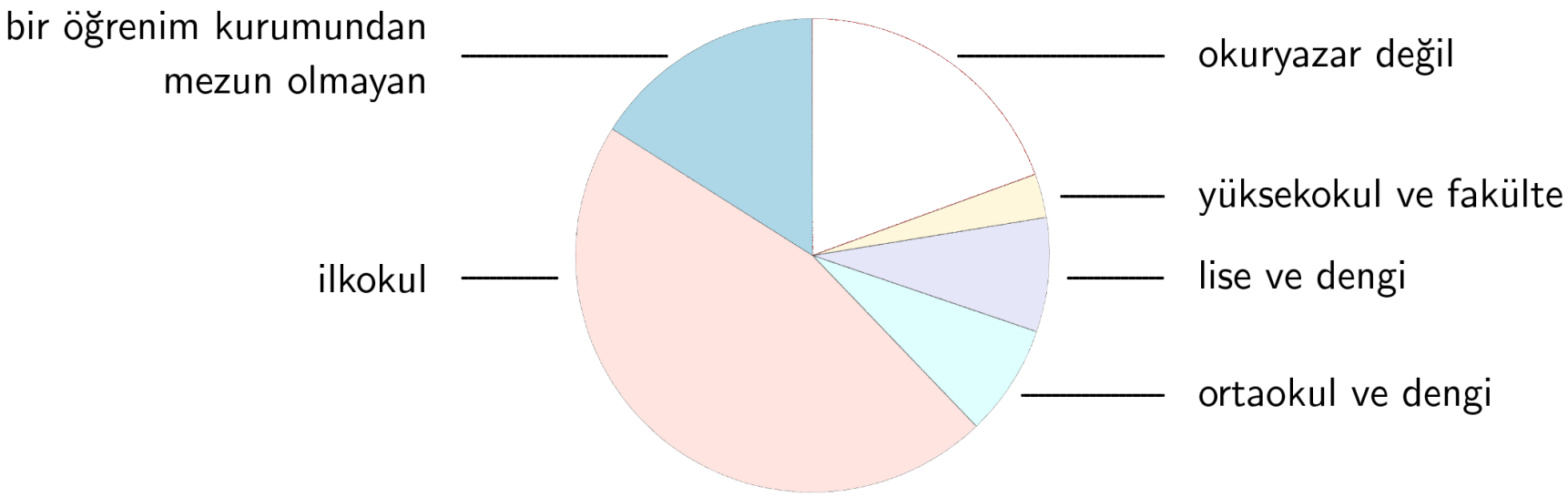
3.1 Frequency Distributions

Example: Educational attainment in Turkey, 1990.

Category		h_i	f_i
1:	okuryazar değil	9.56	0.195
2:	bir öğrenim kurumundan mezun olmayan	7.84	0.160
3:	ilkokul	22.68	0.462
4:	ortaokul ve dengi	3.72	0.076
5:	lise ve dengi	3.82	0.078
6:	yüksekokul ve fakülte	1.50	0.030
	$\sum$	49.14	1.000

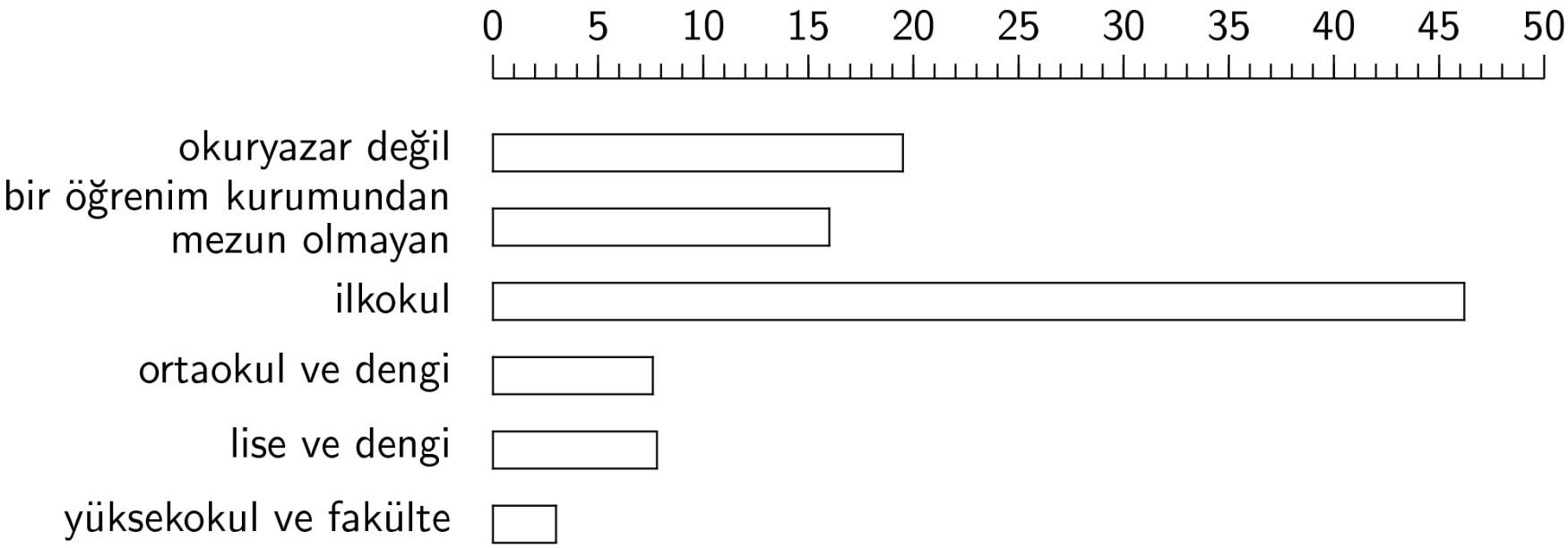
3.1 Frequency Distributions

Example: Educational attainment in Turkey, 1990.



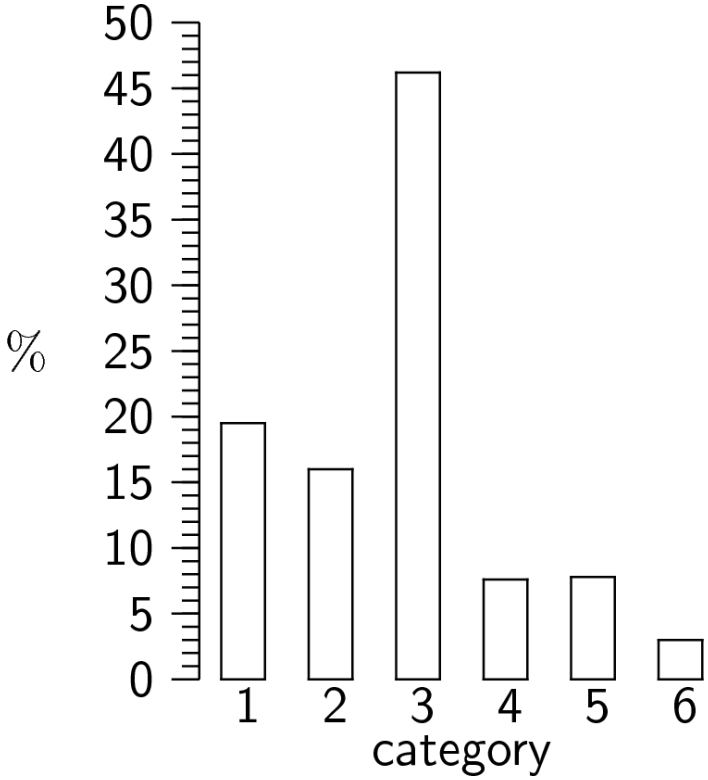
3.1 Frequency Distributions

Example: Educational attainment in Turkey, 1990.



3.1 Frequency Distributions

Example: Educational attainment in Turkey, 1990.



1: okuryazar değil
2: bir öğrenim kurumundan mezun olmayan
3: ilkokul
4: ortaokul ve dengi
5: lise ve dengi
6: yüksekokul ve fakülte

3.1 Frequency Distributions

Example: The number of goals scored in 170 matches of Beşiktaş İstanbul.

Raw data:

4, 8, 2, 3, 2, 3, 4, 2, 2, 1, 2, 5, 5, 4, 0, 2, 6, 2, 2, 3, 1, 4, 4, 5, 4, 4, 10, 3,
3, 1, 3, 2, 0, 0, 2, 7, 6, 2, 1, 2, 5, 2, 4, 1, 0, 3, 4, 3, 3, 5, 4, 4, 0, 3, 0, 1, 7,
4, 3, 3, 5, 6, 4, 2, 7, 5, 4, 2, 3, 4, 3, 0, 3, 3, 3, 1, 0, 5, 1, 3, 2, 8, 4, 6, 3, 2,
4, 2, 1, 4, 1, 5, 5, 3, 1, 3, 2, 5, 4, 2, 1, 0, 5, 3, 2, 1, 6, 2, 3, 4, 5, 1, 2, 3,
2, 2, 2, 2, 0, 1, 3, 2, 2, 3, 3, 2, 3, 3, 2, 1, 0, 3, 2, 3, 3, 4, 1, 4, 2, 6, 1, 4,
3, 0, 2, 5, 1, 1, 4, 1, 3, 3, 2, 4, 2, 2, 3, 3, 3, 2, 1, 3, 4, 4, 2, 4, 6, 6, 4, 5

What can we do with this dataset?

3.1 Frequency Distributions

Example: The number of goals scored in 170 matches of Beşiktaş İstanbul.

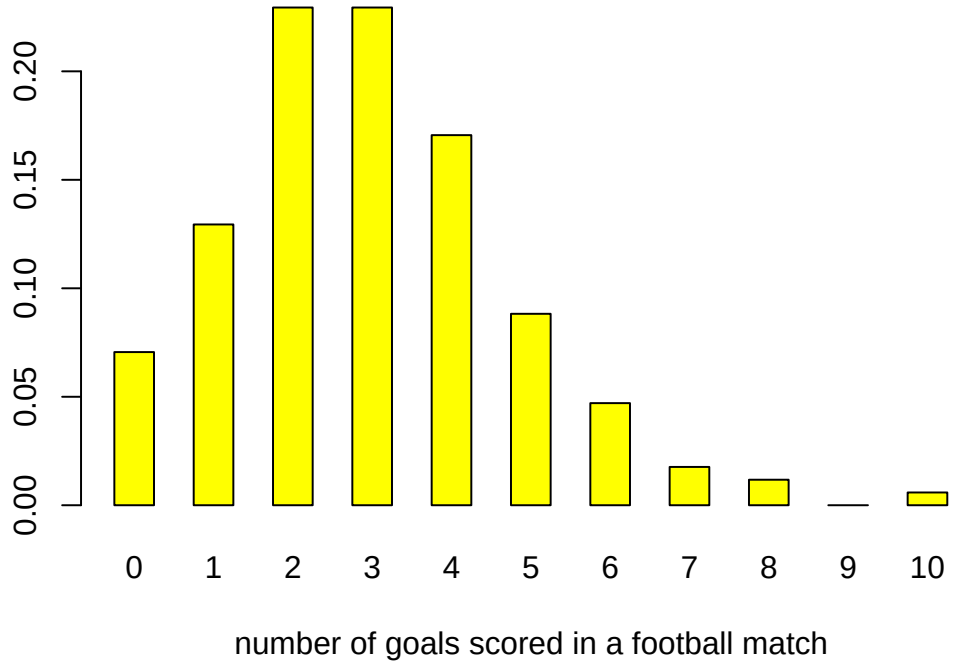
The empirical distribution is:

i	0	1	2	3	4	5	6	7	8	9	10
h_i	12	22	39	39	29	15	8	3	2	0	1
f_i	$\frac{12}{170}$	$\frac{22}{170}$	$\frac{39}{170}$	$\frac{39}{170}$	$\frac{29}{170}$	$\frac{15}{170}$	$\frac{8}{170}$	$\frac{3}{170}$	$\frac{2}{170}$	$\frac{0}{170}$	$\frac{1}{170}$

3.1 Frequency Distributions

Example: The number of goals scored in 170 matches of Beşiktaş İstanbul.

A bar chart of the distribution:



3.1 Frequency Distributions

Example: The number of goals scored in 170 matches of Beşiktaş İstanbul.

How can we compute the average number $\bar{x}$ of goals per match?

- Use the observations $x_1, \ldots, x_{170}$ themselves:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{170}(4 + 8 + \ldots + 5) = 2.96$$

- Use the distribution of the observations:

$$\bar{x} = \sum_i i \cdot f_i = 0 \cdot \frac{12}{170} + 1 \cdot \frac{22}{170} + \ldots 10 \cdot \frac{1}{170} = 2.96$$

3.2 Stemplots and Histograms

Pie charts and bar charts are not suitable to display the distribution of a continuous metric variable, such as:

X = total expenditure of a customer at a supermarket (when shopping once),

X = duration of a TV commercial.

For this, we need a stemplot (if the number of observations is not too big) or a histogram.

3.2 Stemplots and Histograms

Example: Duration of a TV commercial.

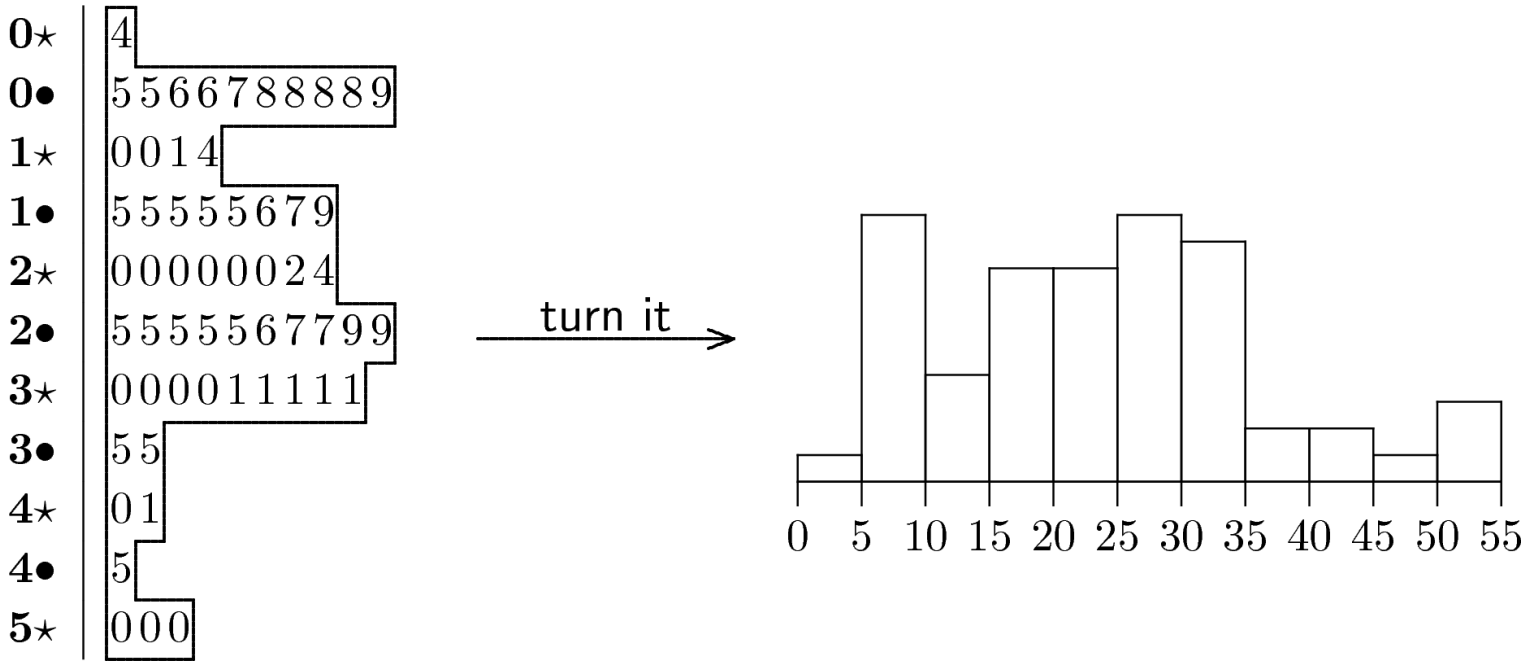
0|4 =4 seconds

Germany			Turkey
4	**0⋆**		
9888876655	**0•**	56678	
4100	**1⋆**	02222222224	
97655555	**1•**	55677778899	
42000000	**2⋆**	0000011222223	
9977655555	**2•**	55677	
111110000	**3⋆**	000002233	
55	**3•**	5557	
10	**4⋆**		
5	**4•**		
000	**5⋆**		
	HI	122, 122	

3.2 Stemplots and Histograms

Example: Duration of a TV commercial.

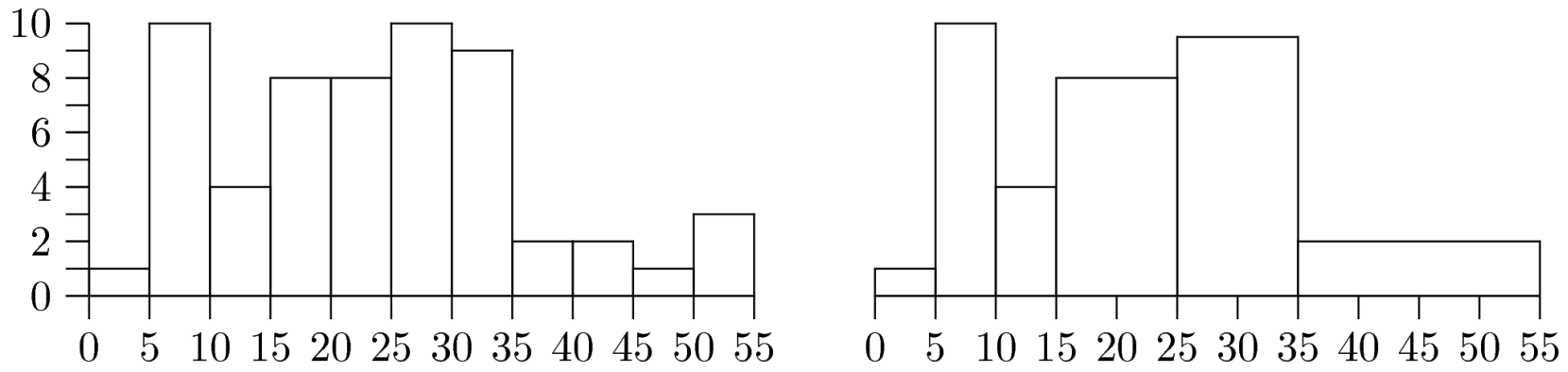
How to obtain a histogram.



3.2 Stemplots and Histograms

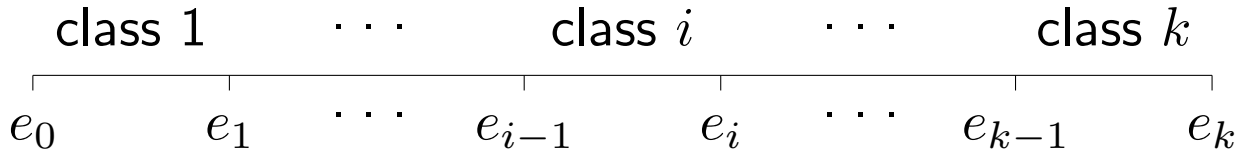
Example: Duration of a TV commercial.

Two versions of a histogram — detailed and less detailed.



3.2 Stemplots and Histograms

Construction of a histogram.



number of classes: k

for each class i $(i = 1, \ldots, k)$:

class limit: lower: e_{i-1}
upper: e_i

class width: $d_i = e_i - e_{i-1}$

A **histogram** consists of rectangles over each class with **area** proportional to the number of observations in each class.

3.2 Stemplots and Histograms

Construction of a histogram.

Let:

- $h_i =$ number of observations in class i
- $H_i =$ height of rectangle over class i

Then:

$$H_i \cdot d_i \propto h_i, \quad \text{or:} \quad H_i = \alpha \cdot \frac{h_i}{d_i}$$

3.2 Stemplots and Histograms

Example: Total expenditure of customers in a supermarket.

The total expenditure of 508 customers (in euros) was recorded.

Raw data: 10.07, 22.61, 14.48, . . . , 28.68
Ordered raw data: 0.59, 0.72, 0.74, . . . , 75.54

How do we get a first insight into this dataset?

3.2 Stemplots and Histograms

Example: Total expenditure of customers in a supermarket.

A (not very detailed) histogram can be obtained like this:

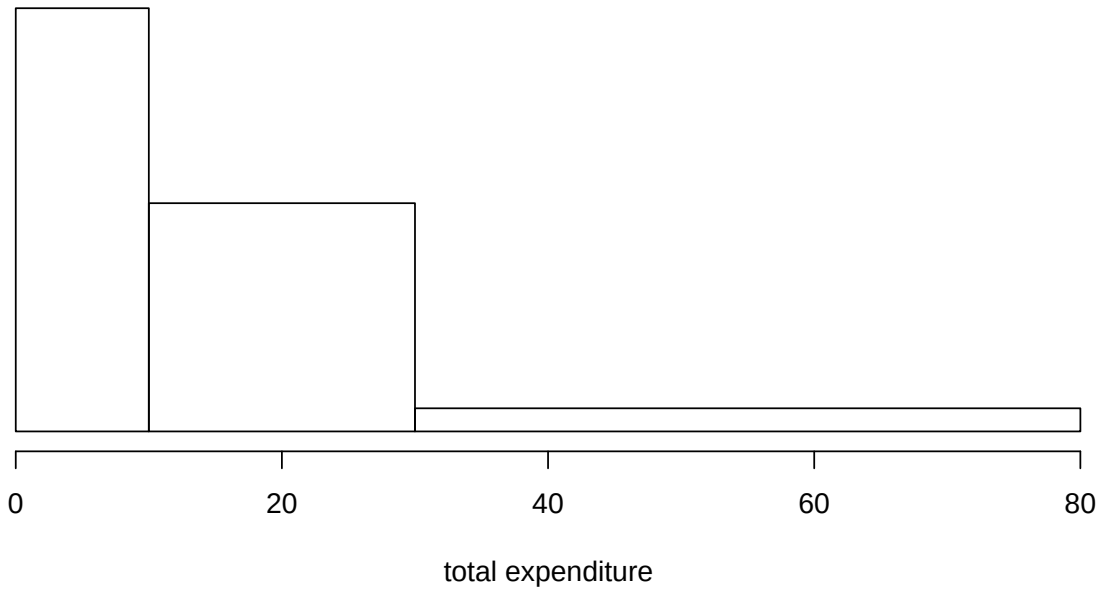
i	interval	h_i	d_i	$\alpha \cdot h_i/d_i$
1	[0, 10)	216	10	21.60α
2	[10, 30)	233	20	11.65α
3	[30, 80]	59	50	1.18α

For a nice picture on an A4 page, choose $\alpha = 0.3$cm (for example).

3.2 Stemplots and Histograms

Example: Total expenditure of customers in a supermarket.

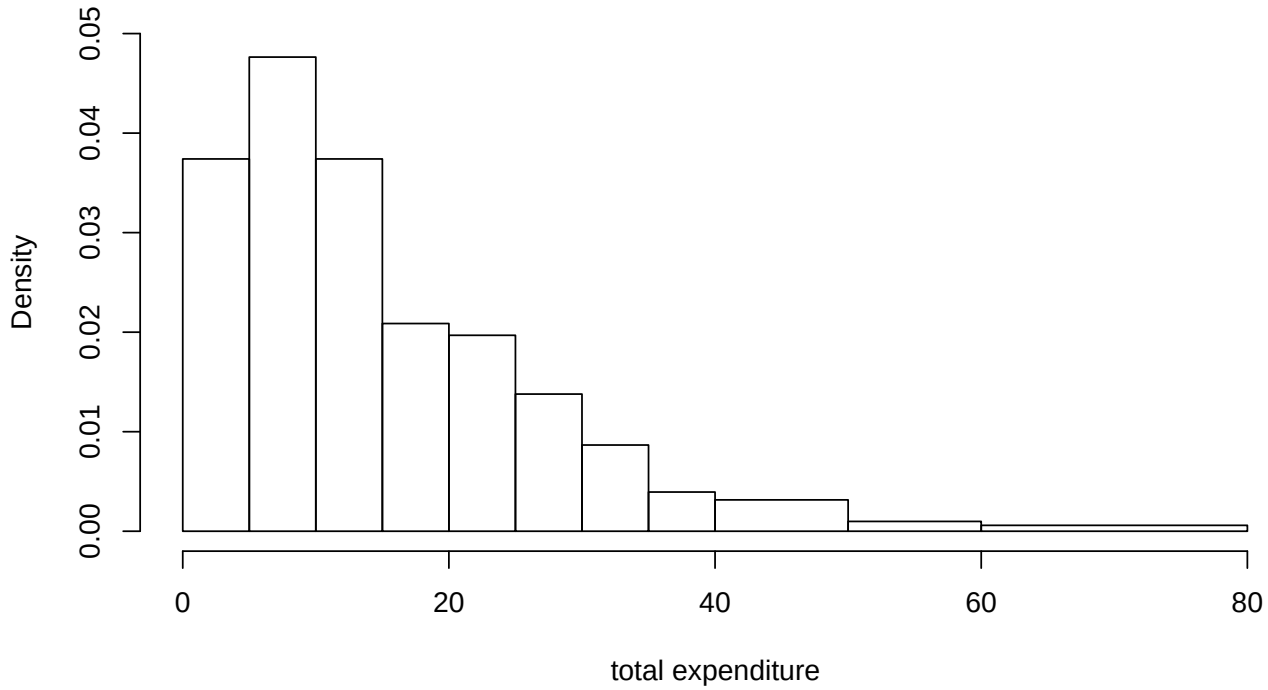
The resulting histogram is:



3.2 Stemplots and Histograms

Example: Total expenditure of customers in a supermarket.

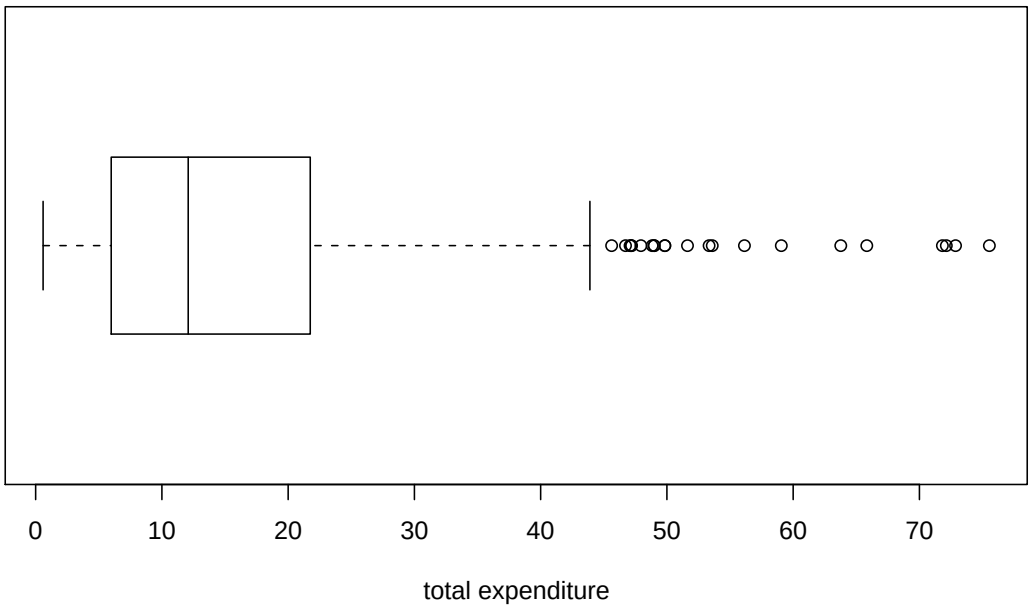
A more detailed histogram is:



3.2 Stemplots and Histograms

Example: Total expenditure of customers in a supermarket.

A boxplot is another elegant way to display the distribution:



3.3 The Shape of a Distribution

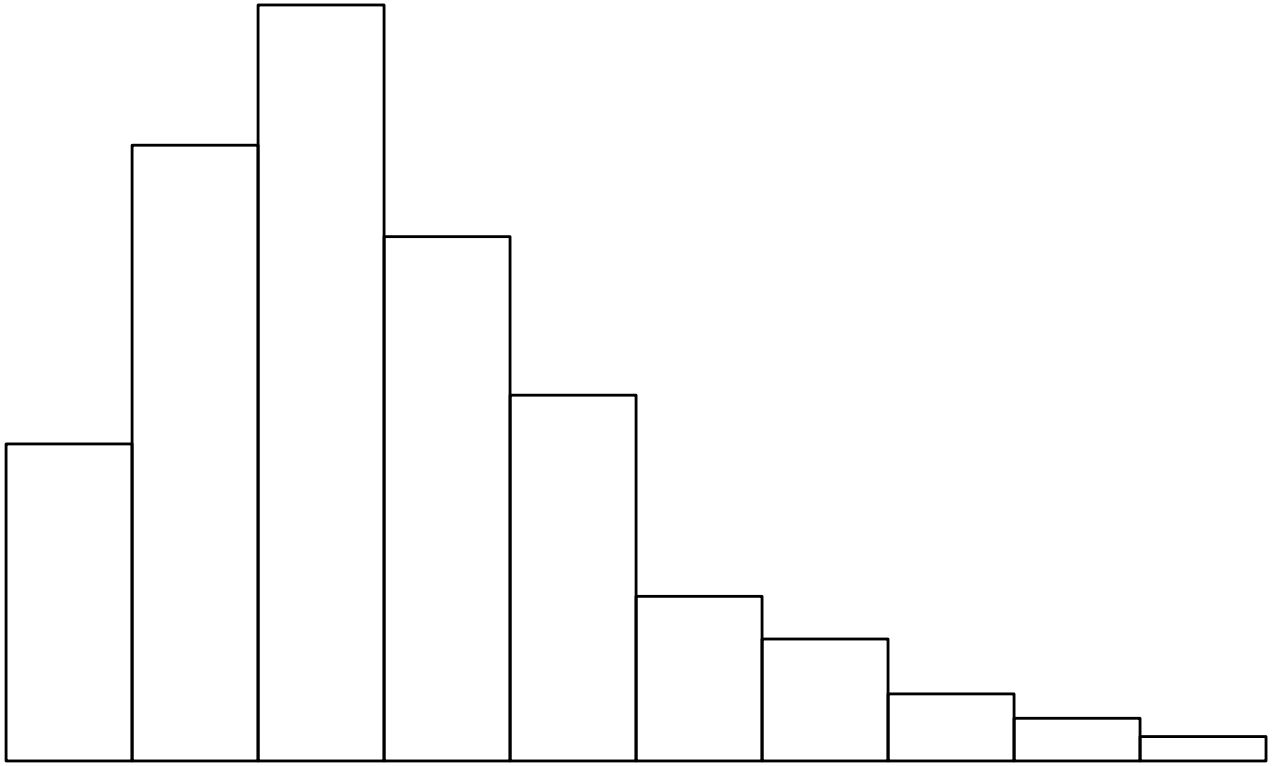
The shape of a distribution.

The shape of a distribution is often an interesting clue. We distinguish distributions with respect to:

- skewness
- kurtosis

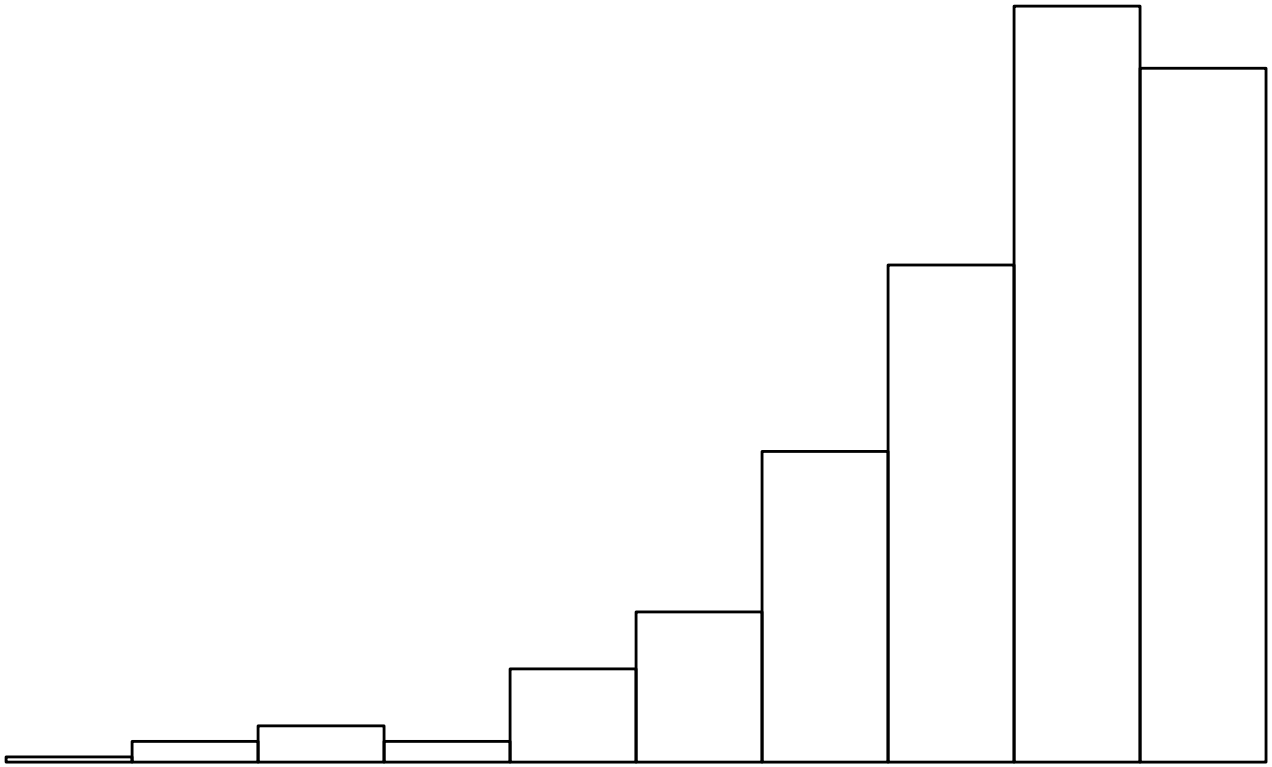
3.3 The Shape of a Distribution

A right-skewed distribution:



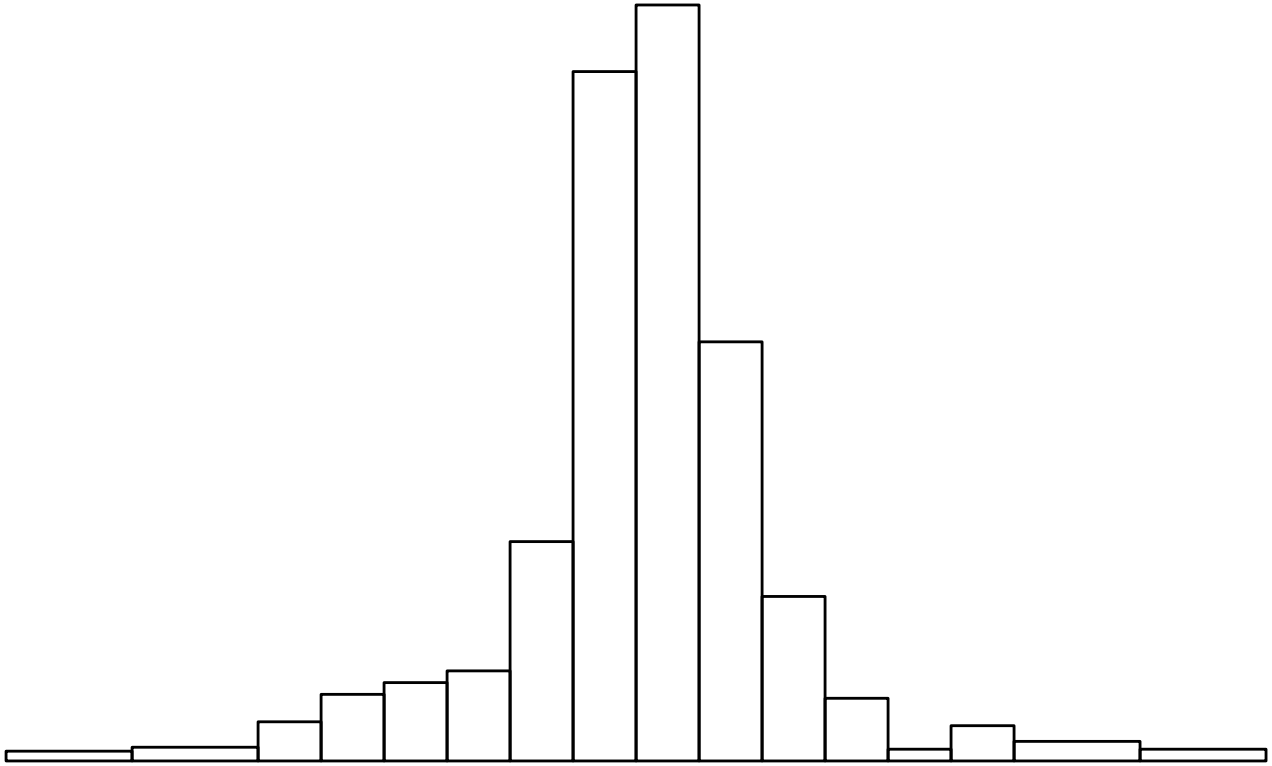
3.3 The Shape of a Distribution

A left-skewed distribution:



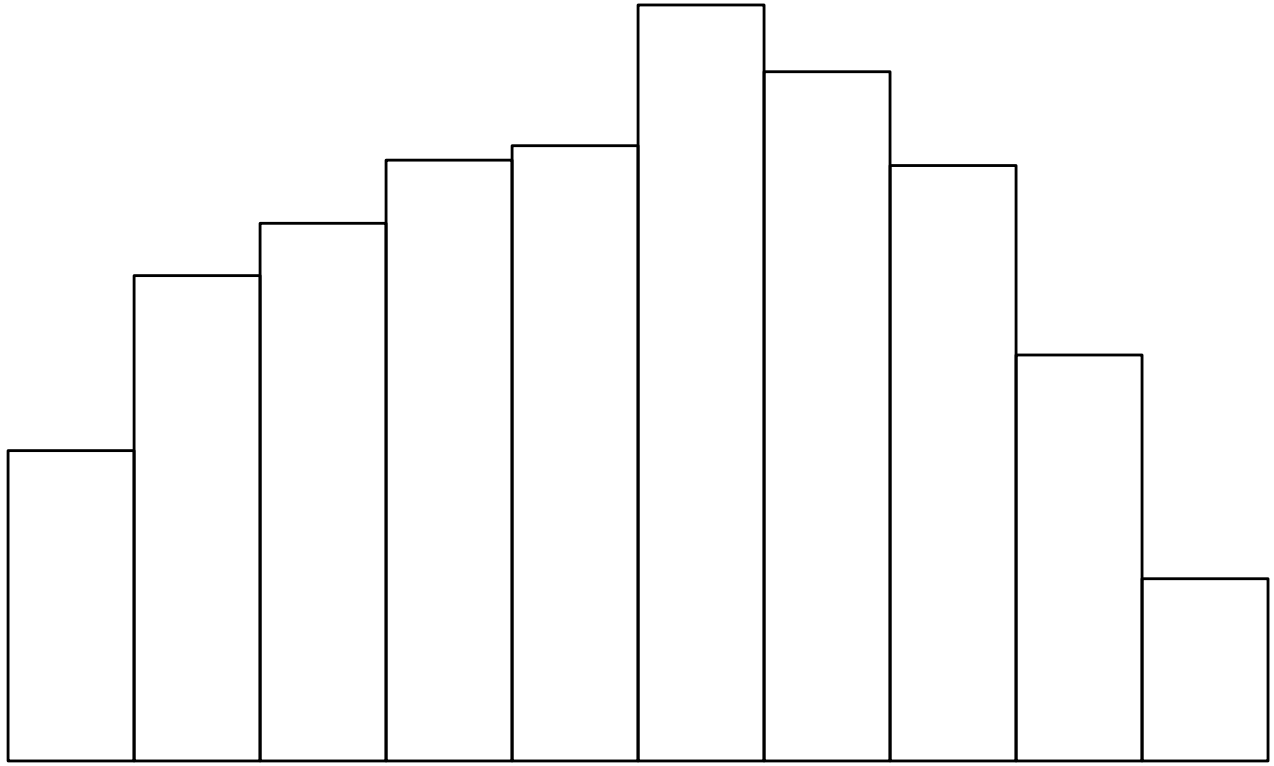
3.3 The Shape of a Distribution

A lepto-kurtic distribution:



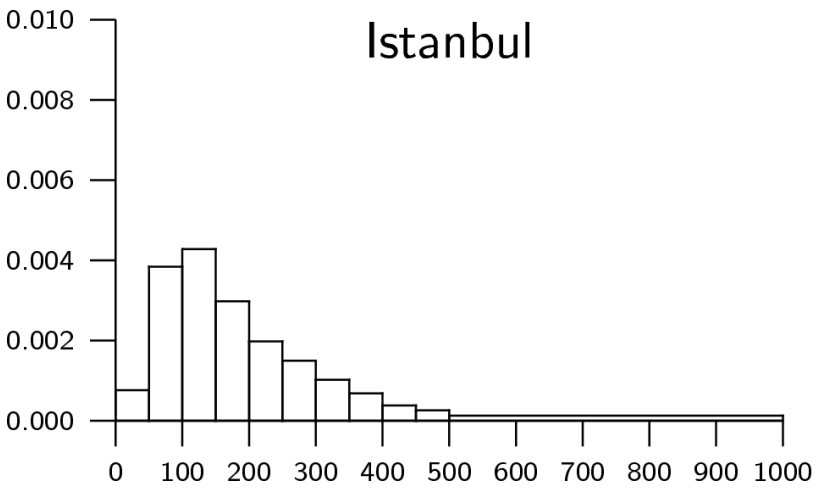
3.3 The Shape of a Distribution

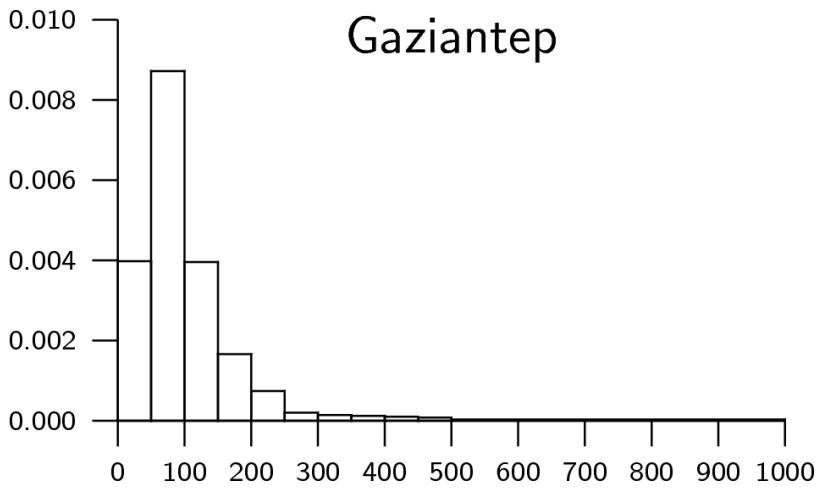
A platy-kurtic distribution:



3.3 The Shape of a Distribution

Example: Household income. (1994; mill. TL)





3.3 The Shape of a Distribution

Example: Monthly, weekly, daily returns on the Dow-Jones Industrial Average, 1995-01 through 2005-10.